Fracture mechanics analysis of adhesive lap joints

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Abstract:

The Rice Cherepanov J is calculated for a lap joint in pure shear. By setting $J=J_c$ the critical value of J integral the fracture load of the joint is calculated for perfectly plastic behavior of the adhesive. The fracture load as function of over-lap length is evaluated both theoretically and experimentally. Three failure modes are given for the joint failure : for small over-lap length, joints fail by adhesive plastification; for middle one, joints fail by cracking and plastification in adhesive; for large one, joints fail by cracking in adhesive.

1 Introduction

For theoretical strength analyses of adhesive joints, different methods from the sciences of structural mechanics and strength of materials may be employed. These methods include linear elastic fracture mechanics[1] and elastoplastic fracture analyses[2,3], elastic and elasto-plastic analyses together with some maximum stress or strain criterion[4].

An experimental device often adopted uses a single or double lap joint loaded in shear. The simplified analysis of Volkersen[5] allows under those conditions, by neglecting the bending of the adherents which remain elastic, to calculate the shear stress in the adhesive. The J-integral can then be calculated and this what we intend to show in order to provide a method of evaluation of the joint resistance to crack propagation.

Here in our paper, the major emphasis is given to nonlinear fracture mechanics analysis and to relate the fracture load as function of the overlap length to different fracture modes.

2 Fracture mechanics approach

A lap joint of length 2l, width B, thickness h is shown in figure 1. The two adherents of thicknesses and Young moduli respectively h_1 and h_2, E_1 and E_2 are considered as linear elastic bars in pure tension and the adhesive is regarded as elastic / perfectly plastic pure shear medium. A force F is applied on each arm.

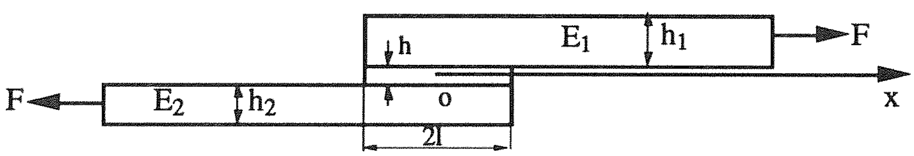

figure 1 Sketch of a lap joint

Choosing the center of the joint as the origin of the coordinate, we obtain easily the following basic equations:

$$\tau = h_{1s}\frac{d\sigma_1}{dx} = -\,h_{2s}\frac{d\sigma_2}{dx} \quad (1)$$

$$\sigma_1 = E_{1s}\frac{du_1}{dx} \quad (2)$$

$$\sigma_2 = E_{2s}\frac{du_2}{dx} \quad (3)$$

$$\sigma_1 h_{1s} + \sigma_2 h_{2s} = \frac{F}{w} \quad (4)$$

We consider a crack starting at one end of the joint (figure 2). A contour ABCDF along the joint is chosen as shown in figure 2.

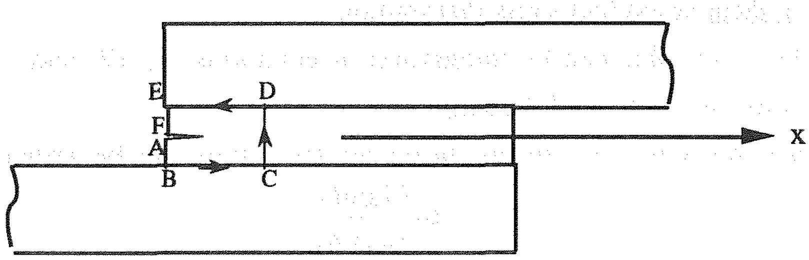

figure 2 Contour for the calculation of the J integral

The Rice Cherepanov J is calculated by:

$$J=\int_{ABCDEF} W(x)dy-(t_x\frac{du_x}{dx}+t_y\frac{du_y}{dy})ds \tag{5}$$

where W(x) is the strain energy density, t_x and t_y are the components of the stress on the contour, and u_x and u_y those of the displacement.

It reduces to

$$J=-\int_{BC} t_x\frac{du_x}{dx}dx+\int_{CD} W(x)dy-\int_{DE} t_x\frac{du_x}{dx}(-dx) \tag{6}$$

or

$$J=\int_{BC} \tau(x)\frac{\sigma_2}{E_2}dx+W(x)h+\int_{DE} \tau(x)\frac{\sigma_1}{E_1}dx \tag{7}$$

because

$$\frac{du_1}{dx}=\frac{\sigma_1}{E_1} \text{ and } \frac{du_2}{dx}=\frac{\sigma_2}{E_2} \tag{8}$$

Using the relations (1) and the boundary conditions $\sigma_2(-l/2)=F/Bh_2$ and $\sigma_1(-l/2)=0$, the preceding integral (7) yields:

$$J=\frac{1}{2}(\frac{F}{B})^2\frac{1}{E_2h_2}-\frac{1}{2}\frac{h_2}{E_2}\sigma_2^2-\frac{1}{2}\frac{h_1}{E_1}\sigma_1^2+W(x)h \tag{9}$$

It is easy to demonstrate that this integral is contour independent whatever the mechanical behavior of the adhesive, by showing that dJ/dx=0. This results from the following:

$$W=\int_0^{\tau} \tau d\gamma \text{ and } \frac{dW}{dx}=\tau\frac{d\gamma}{dx}=\frac{\tau}{h}\frac{d(u_1-u_2)}{dx}=\frac{\tau}{h}(\frac{\sigma_1}{E_1}-\frac{\sigma_2}{E_2}) \tag{10}$$

3° Symmetrical case with the perfectly plastic adhesive

We will consider the case where the two adherents are identical with two symmetric cracks at each end of the joint leaving 2b of the undamaged adhesive.

$$h_1=h_2=h_s \qquad E_1=E_2=E_s$$

The calculation of the stress should be made using the effective overlap length as the stress relaxed in the cracked region.

3.1 shear stress and strain distribution

The result obtained by integration of equations (1), (2) and (3) is well known [6,7], which is summarized in the following:

when the adhesive remains elastic, the shear stress can be written

$$\tau=\frac{F\lambda ch(\lambda x)}{2wsh(b\lambda)} \qquad (11)$$

where

$$\lambda=\sqrt{\frac{2G_c}{hE_sh_s}}$$

The applied force corresponding to adhesive yield is obtained by setting $\tau=\tau_y$ for x=b, yields

$$F_y=\frac{2\tau_y}{\lambda}th(b\lambda) \qquad (12)$$

The shear strain in the adhesive can be easily obtained using the adhesive constitutive equation.

When $F>F_y$, plastic zones will spread from joint ends. Assuming that x_p is the coordinate of plastic zone head from joint center (figure 3), In the plastic zone ($x>x_p$), the shear stress remains constant:

$$\tau=\tau_y \qquad x_p\leq x\leq b \qquad (13)$$

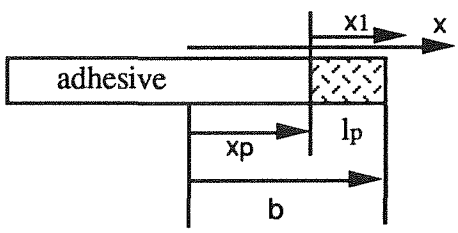

figure 3 Local reference frame with plastic zone

In the elastic region, the shear stress is obtained from the boundary condition $\tau=\tau_y$; $x=x_p$:

$$\tau=\frac{\tau_y ch(\lambda x)}{ch(\lambda x_p)} \qquad 0\leq x\leq x_p \qquad (14)$$

x_p is determined using the equilibrium equation of the joint:

$$\frac{\tau_y th(\lambda(b-l_p))}{\lambda}+\tau_y l_p=\frac{F}{2w} \qquad (15)$$

where $l_p=b-x_p$

Equations (13), (14) and (15) give the shear stress distribution in a plastic joint.

Shear strain in elastic region is straightforward. In the plastic zone, the shear stress remains constant, but the shear strain in the adhesive is constrained by the adherents. The shear strain in the adhesive is obtained [7] after the calculation of the strain in the adherents :

$$\gamma = \frac{\tau_y}{G_c} + \left(\frac{Fx_1}{wh_sE_sh} - \frac{\tau_y(2l_px_1-x_1^2)}{hh_sE_s} \right) \tag{16}$$

where x_1 is the coordinate with the origin at the head of the plastic zone (figure 3).

3.2 calculation of integral J

A special enclosed path along the plastic zone is used for calculating the J integral. Using the shear stress and strain distribution determined previously , equation (9) gives:

$$J=\frac{\tau_y^2}{2G_c}h + \frac{F\tau_y l_p}{wh_sE_s} \frac{\tau_y^2 l_p^2}{h_sE_s} \tag{17}$$

when lp=l, the adhesive is totally plastic, equation (17) ceases to be valid, the failure load of the joint is simply equal to the limit load.

The J-integral and the plastic zone size as function of the adhesive thickness is shown in figure 4 for E_s=7300MPa, G_c=712MPa, h_s=5.75mm, τ_y=24MPa, l=25mm, l-b=0.5mm and under applied force F/w=0.75MN/m .

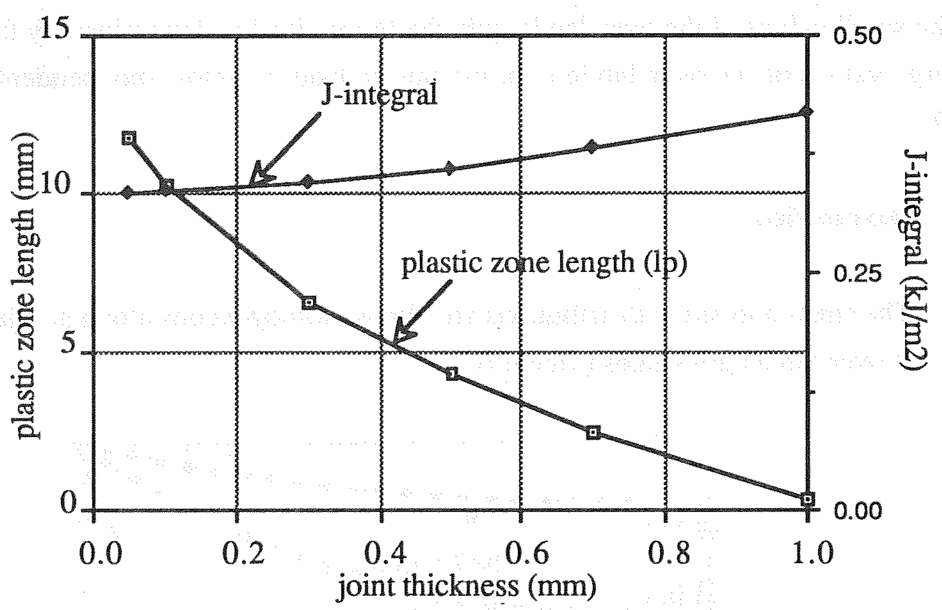

figure 4 Variation of J-integral and plastic zone length as function of over-lap length

For a constant applied force, the plastic zone is larger and J-integral is smaller for thinner joints. If we apply the criterion $J=J_c$ to determine the joint strength, It is found that the joint strength decreases with the adhesive thickness, but for the large over-lap length values, J is less sensible to the adhesive thickness [8].

The joint strength as function of the over-lap length is shown in figure 5 (J_C=0.33kJ/m, l-b=h=0.5mm, and the other parameters remain unchanged).

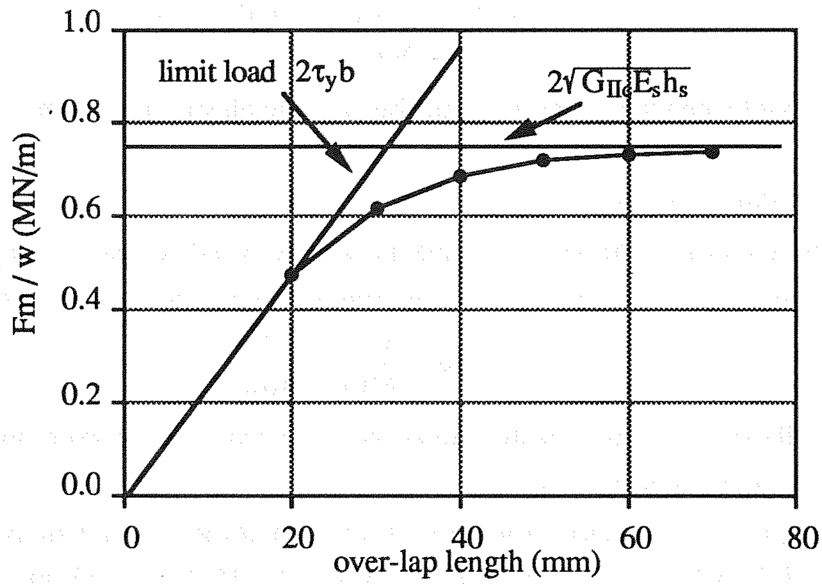

figure 5 Joint strength variation as function of over-lap length

For the small values of the over-lap length, the failure load is determined by the limit load; for the large values of the over-lap length, the failure load is almost independent of the over-lap length.

4 Discussion

The stress and strain distributions are almost homogeneous along all the joint length for the small over-lap length values (figure 6).

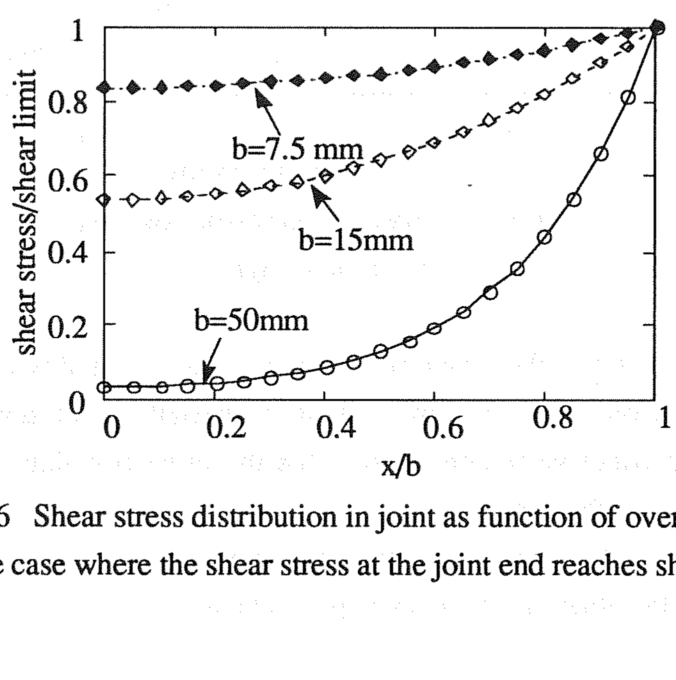

figure 6 Shear stress distribution in joint as function of over-lap length
(for the case where the shear stress at the joint end reaches shear limit)

In this case, shear stress or shear strain reaches ultimate value almost simultaneously, therefor failure load is determined by plastification of the adhesive, it is proportional to the over-lap length of the joint ($2\tau_y b$).

For the large values of the over-lap length, the concentrated shear strain zone is localized near the ends of joint. Adhesive plastification and crack initiation may occur at about the same moment because of the high concentrated shear strain. In such case, the plastic zone can be neglected owing to crack presence. Joint strength is governed only by the cracking resistance in the adhesive and the linear fracture mechanics can be applied. We choose a half joint (figure 7), a crack of the length a is taken into account by increasing upper adherent length and reducing the lower one.

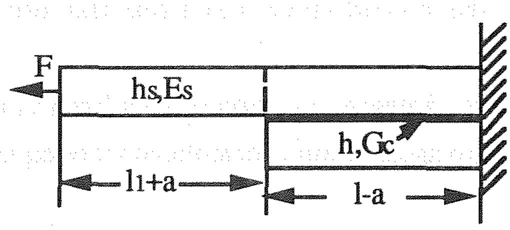

figure 7 model of joint cracking

Joint compliance can be written as follws :

$$c = \frac{(l_1+a)}{E_s w h_s} + \frac{(l-a)}{E_{eff} w h_{eff}} \quad (18)$$

where E_{eff} and h_{eff} are the modulus and the thickness of non cracking part of the joint.

$$E_{eff} = (\frac{2h_sE_s}{2h_s+h} + \frac{hE_c}{2h_s+h})$$
$$\approx E_s \quad (19)$$

$$h_{eff}=2h_s+h$$
$$\approx 2h_s \quad (20)$$

Using the relation between the compliance and the energy release rate in the case of the linear fracture mechanics, we have:

$$G=\frac{F^2}{2w}\frac{\partial c}{\partial a}$$
$$=\frac{F^2}{4w^2E_sh_s} \quad (21)$$

The joint strength is obtained by setting the criterion $G=G_{IIc}$:

$$\frac{F_m}{w}=2\sqrt{G_{IIc}E_sh_s} \quad (22)$$

Obviously from equation (22), the joint strength is independent of the over-lap length, and it increases with the critical energy release rate of the adhesive. This result can be found as a special case of the J integral approach for the large values of the over-lap length. In fact, for the large values of the over-lap length, equation (15) becomes:

$$\frac{F}{2w} \approx \tau_y l_p + \frac{\tau_y}{\lambda} \tag{23}$$

Replacing l_p obtained from equation (23) to equation (17), we obtain for J-integral:

$$J = \frac{F^2}{4w^2 E_s h_s}$$

The same expression for G and J is found for large over-lap joints.

For the middle values of the over-lap length, joint strength is determined by cracking and plastification in the adhesive, and the nonlinear fracture mechanics needs to be used.

Nonlinear fracture mechanics approach gives an unified approach for the different joint fracture modes including plastification, plastification and cracking and cracking of the adhesive joints. The joint strength is determined, in our approach, by two mechanical parameters of the adhesive (shear modulus, shear yield stress) and one fracture parameter (critical energy released rate Jc).

It should be noted that J-integral criterion differs from those of maximum shear stress and strain. In fact the maximum shear strain is calculated from equation (16) by setting $x_1 = l_p$:

$$\gamma_{max} = \frac{\tau_y}{G_c} + \left(\frac{F l_p}{w h_s E_s h} - \frac{\tau_y l_p l_p^2}{h h_s E_s} \right) \tag{24}$$

the following relation between maximum shear strain and J-integral is found:

$$J = h\tau_y(\gamma_{max} - \frac{\gamma_e}{2}) \tag{25}$$

where γ_e is the elastic shear strain of the adhesive.

5 Experimental analysis

Experiments are carried out to validate the preceding analyses. Double-lap equilibrium joints are examined for two epoxy adhesives (ECCOBOND45LV, ESP110). The adherent is aluminium alloy with modulus 70000MPa and the characteristics of the adhesives are shown in figure (8).

The comparison results are shown in figure (8).

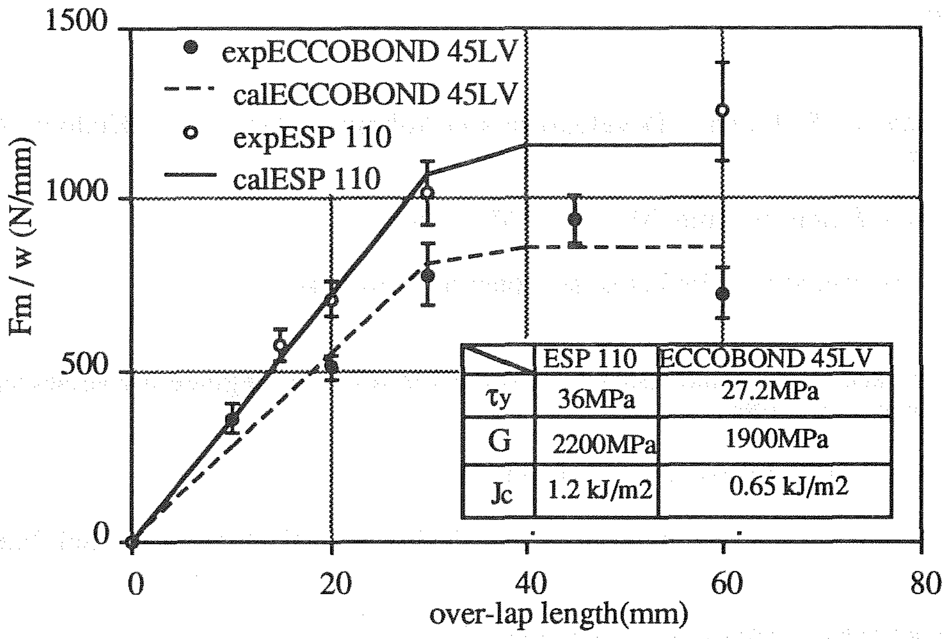

figure 8 Comparison between experiment and modelling

A good agreement between the experiments and the modelling is obtained.

6 Conclusion

Three failure modes can explain the joint strength variation as function of over-lap length: for the small values of the over-lap length, joints fail by adhesive plastification; for the middle ones, joints fail by plastification and cracking in the adhesive, and for the large ones, joints fail by cracking. The non-linear fracture mechanics method can offer an unified approach for these three types of joint failure. The joint strength is determined completely by two mechanical parameters of the adhesive (shear modulus and shear yield stress) and one fracture parameter (critical energy released rate Jc). The experimental results confirmed the present analyses. An advantage of this closed form analyses is that parametric studies can be easily made for engineering design.

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