ZERO-THICKNESS INTERFACE ELEMENTS FOR HYDRAULIC FRACTURE SIMULATION

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Abstract. This paper deals with hydromechanical coupling analysis, paying special attention to the modeling of the Hydraulic Fracture phenomenon. The problem is tackled using a new fully integrated approach, which formulates the fracture behavior by means of the same double nodded zero-thickness interface elements for both the mechanical and flow analysis. The mechanical behavior of the joint is reproduced by means of a non-linear constitutive law with fracture work softening. An innovative flow model is developed for the joint, since it reproduces both longitudinal and normal flows through a double-nodded zero-thickness interface element, resulting in an improvement of the existing double-nodded models. The coupled problem is solved following a staggered strategy, where the same FE mesh is used. The coupling terms and mechanisms are also described and formulated.

Resumen. El presente artículo se sitúa en el campo de los problemas hidro-mecánicos, y particularmente en el estudio del fenómeno de la Fractura Hidráulica. Se plantea la resolución numérica de este problema mediante la utilización de un mismo elemento junta para representar el comportamiento mecánico e hidráulico de la discontinuidad, lo que supone un enfoque nuevo y global del problema. Para reproducir el comportamiento mecánico de la junta se utiliza una ley no lineal con reblandecimiento por trabajo de fractura. Por su parte, el modelo de flujo para la junta representa una innovación dentro de este ámbito de estudio, dado que reproduce ambos flujos, longitudinal y transversal, utilizando un elemento junta de espesor nulo y con nudos duplicados. El problema acoplado se resuelve en forma de "staggered" y utilizando una misma malla de Elementos Finitos. Finalmente se formulan y describen los mecanismos y términos de acoplamiento.

1. INTRODUCTION

In geomechanical or geotechnical problems, the mechanical and hydraulic behavior often appear combined and influenced reciprocally, leading to the so-called hydromechanical coupled problems. The creation and progressive opening of discontinuities, as fissures or joints, is a key aspect in these problems, and they may represent an additional coupling factor. These discontinuities may play a very important role within the structures safety and performance, oil exploitation processes and aquifers performance, among others. The hydraulic fracture phenomenon is a classical example within the hydromechanical coupling field involving the presence of discontinuities.

The problem of hydraulic fracture has traditionally been studied within the oil exploitation and recovery field, as well as within the evaluation of in-situ stresses in rock masses. In both cases, after a well is drilled, a fluid is pumped down the well at high pressures, so that the fluid runs inside the intercepted discontinuities and causes the joints to open and propagate. The flow rate and pressure measures may be afterwards related to some theoretical model [1-3].

During the last two decades more complex numerical models have been developed. These more recent analyses are using a progressively better discretization of the domain and description of the processes involved in the problem, as well as of its interaction. Within this field, we may emphasize the work done by Prof. Ingraffea at Cornell University [4,5], who has developed a model to tackle the coupled problem in a global fashion. This model combines a Finite Element Method formulation for the mechanical problem, with Finite Differences for the flow mesh.

However, the hydraulic fracture phenomenon and other similar problems with coupling through interfaces may be efficiently tackled just by means of the FEM. In this article, the on-going work to develop a new fully integrated approach is presented, which uses the same double nodded zero-thickness interface element to model the mechanical and the hydraulic behavior of a joint. In this way, we can use the same FE mesh for both analyses, which may be very advantageous in the context of a staggered strategy. Sect.2 of the article briefly refers to zero-thickness interface element models used for the mechanical analysis, which have been well established since some time already (see for instance [8-10]). Special attention is given to the analysis of the hydraulic behavior of joints in Sect.3, where we describe a recently proposed interface model for flow analysis through discontinuities [6]. This FE interface model makes use of double-nodded zero-thickness elements, which results in an innovation since the existing models are based on either single or triple nodded zero-thickness interface elements. Sec.4 includes a description of the coupling mechanisms and terms that exist between both the mechanical and the hydraulic problems when using a staggered procedure. This gives an idea of the steps involved in a coupled calculation such as the hydraulic fracture analysis. Finally, some concluding remarks are given in Sect. 5.

2. INTERFACE ELEMENTS FOR MECHANICAL ANALYSIS.

The approach used for mechanical fracture analysis follows the so-called discrete crack approach [7], where every discontinuity is represented individually, and can be integrated into the FEM consistently by means of zero-thickness interface elements with double nodes, which are equipped with appropriate constitutive laws [8].

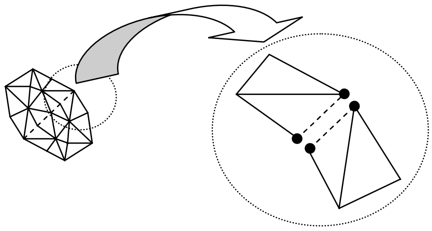

Fig. 1. Zero-thickness interface element with double nodes inserted into the Continuum FE mesh.

Interface behavior is reproduced by means of a work-softening elasto-plastic constitutive law, and it is formulated in terms of the stresses on the interface plane $\boldsymbol{\sigma} = [\sigma_n \quad \sigma_t]^{\mathrm{T}}$, normal and shears, and the corresponding normal and tangential relative displacements $\mathbf{u} = [u_n \quad u_t]^{\mathrm{T}}$. The interface constitutive model is that described and analyzed in detail in [8], and used in many other analyses [9,10] involving crack opening and propagation.

3. INTERFACE ELEMENTS FOR FLOW ANALYSIS

Fluid flow through discontinuities has traditionally been modeled using special elements of zero-thickness, which we can classify into single, double and triple nodded. Single-node elements are the simplest and most commonly used for instance in geohydrology. They consist of "line" or "pipe" elements which are superimposed on the standard continuum element edges using the same node common to adjacent elements. These elements can only model the longitudinal flow through the fracture with a hydraulic conductivity K_l. On the other hand, some authors have included the transversal flow with its transmissivity K_t, and the subsequent localized potential drop, by using triple-node interface elements. In those, the two nodes of the adjacent continuum elements represent the potentials in the pore system on each side of the interface, and a third node in the middle represents the average potential of the fluid in the channel represented by the discontinuity [11]. Finally, some authors [12] have proposed double-node interface elements although without considering the influence of the transversal flow, which is equivalent in practice to consider an infinite K_t. These elements are double-nodded only from the geometrical viewpoint, but when time comes to solve the system the two nodes must have the same potential, which can only be obtained by the "trick" of prescribing the equivalence of these two d.o.f. before solving the global system equations.

As already mentioned, fluid flow in a discontinuity may occur in the longitudinal and normal directions. The interface element described in the following includes the two types of flow in the context of a double-nodded geometry. The equations describing both normal and transverse fluid flows are obtained next. The fundamental behavior is postulated in the mid-plane of the joint, and it is incorporated into the interface element when performing the Finite Element formulation. In order to give a more understandable formulation some information regarding parameter units is offered. More details about the element formulation and examples can be found in [6].

3.1. Flow Equations in Interface Elements

Let us consider a differential element of a discontinuity and the existence of some leak-off (q^- and q^+ [L/T]) incoming to the fracture from the surrounding continuum medium.

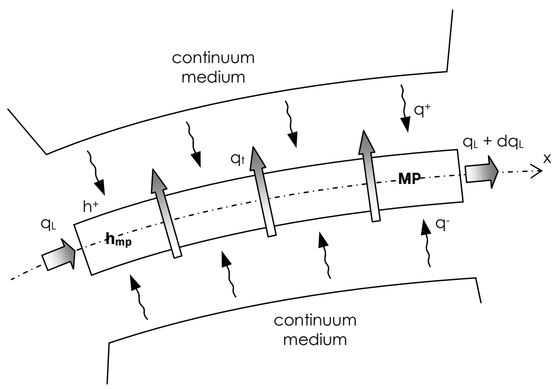

Fig. 2. Flow through the differential joint element.

We assume that we are dealing with a uniform, incompressible and Newtonian viscous fluid.

3.1.1. Longitudinal Flow

By imposing conservation of fluid mass in the longitudinal direction, we get the following 1D continuity equation:

$$-\frac{d\hat{q}_l}{dx}+s+q^-+q^+=\frac{1}{\rho}\frac{\partial}{\partial t}\left(\rho\cdot w\right) \qquad (1)$$

where $\hat{q}_l$ [L^2/T] is the longitudinal local flow rate, ρ is the fluid density, w is the local discontinuity width, i.e. aperture, and s [L/T] is a source term. The second fundamental equation is derived from Darcy's law, which in the particular case of 1D fluid flow leads to:

$$\hat{q}_l = w\cdot q_l = -w\cdot\hat{k}_l\frac{dh_{mp}}{dx} = -k_l\frac{dh_{mp}}{dx} \qquad (2)$$

where h_{mp} is the total head at the mid-plane of the fracture, q_l [L/T] is the fluid flux and $\hat{k}_l$ [L/T] is the longitudinal joint hydraulic conductivity

Substitution of equation (2) into equation (1) leads up to the partial differential equation governing longitudinal fluid flow along the mid-plane of a discontinuity:

$$\frac{d}{dx}\left(k_l\frac{dh_{mp}}{dx}\right)+s+q^-+q^+=\frac{1}{\rho}\frac{\partial}{\partial t}\left(\rho\cdot w\right) \qquad (3)$$

which is analogous to the PDE governing fluid flow through porous medium.

3.1.2. Transversal Flow

The discontinuity may represent an obstacle for the flow in the transversal direction (i.e. it may cause a potential drop due to the transition from a pore system into an open channel and back into a pore system), and therefore we consider a jump in the total head field across the joint $\Delta h_{Tmp} = h_{mp}^- - h_{mp}^+$ associated to the transverse flow q_t [L/T]. We consider a law similar to that of Darcy, but now the total head jump plays the role of the total head gradient:

$$q_t = k_t\cdot\Delta h_{Tmp} \qquad (4)$$

where k_t [1/T] is a transverse hydraulic conductivity coefficient.

3.2. Interface Finite Element Formulation

In the present analysis, for simplicity, we shall develop the formulation for a linear zero-thickness interface element. Since the governing equations have been formulated in the joint mid-plane, we are going to define two virtual nodes on the interface mid-plane and in between two facing nodes.

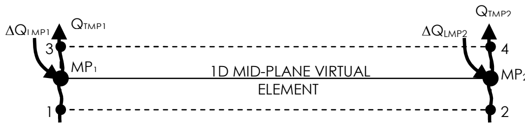

Fig. 3. Definition of the mid-plane element

This procedure considers the mid-plane as a 1D element. The longitudinal and transversal flows are next formulated on this virtual element by means of the FEM. In contrast with existing triple-node formulations, these virtual nodes will be finally eliminated and replaced by the corresponding double-nodes, leading to the actual interface FEM formulation.

3.2.1. Longitudinal Flow

According to the FEM procedure, the main variable in equation (3), h_{mp}, is interpolated within the joint mid-plane by means of its value at the mid-plane nodes:

$$h_{mp}=\sum_{i=1}^{2}N_{mp_i}h_{mp_i}=\mathbf{N}_{\mathrm{mp}}^{\mathrm{T}}\mathbf{h}_{\mathrm{mp}} \qquad (5)$$

where $\mathbf{N}_{\mathrm{mp}}^{\mathrm{T}}=\left[N_{mp_1}\quad N_{mp_2}\right]$ and $\mathbf{h}_{\mathrm{mp}}^{\mathrm{T}}=\left[h_{mp_1}\quad h_{mp_2}\right]$. If we consider no source term, i.e. s = 0, the standard Galerkin method applied to equation (3) gives the following equation governing the longitudinal flow along the mid-plane virtual element:

$$\mathbf{Q}_{\mathrm{int}}=\Delta\mathbf{Q}_{\mathrm{Lmp}}+\dot{\mathbf{Q}}_{\mathrm{mp}} \qquad (6)$$

where

$$\mathbf{Q}_{\mathrm{int}}=\mathbf{Q}_{\mathrm{mp}}^{-}+\mathbf{Q}_{\mathrm{mp}}^{+}=\int_L q^-\mathbf{N}_{\mathrm{mp}}dx+\int_L q^+\mathbf{N}_{\mathrm{mp}}dx \qquad (7)$$

$$\mathbf{\Delta Q}_{\mathrm{Lmp}}=\mathbf{K}_{\mathrm{Lmp}}\mathbf{h}_{\mathrm{mp}} \qquad (8)$$

$$\dot{\mathbf{Q}}_{\mathrm{mp}}=\int_L \dot{w}\mathbf{N}_{\mathrm{mp}}dx \qquad (9)$$

The matrix $\mathbf{K}_{\mathrm{Lmp}}$ is the mid-plane element stiffness matrix for the longitudinal flow, and its expression is:

$$\mathbf{K}_{\mathrm{Lmp}}=\int_L k_l\mathbf{B}\mathbf{B}^{\mathrm{T}}dx \qquad (10)$$

$$\mathbf{B}^{\mathrm{T}}=\left[\frac{\partial N_{mp_1}}{\partial x}\quad\frac{\partial N_{mp_2}}{\partial x}\right] \qquad (11)$$

3.2.2. Transversal Flow

An analogous procedure, but now applied to equation (4), leads up to the following equation governing the transverse flow across the virtual element:

$$\mathbf{Q}_{\mathrm{Tmp}} = \mathbf{K}_{\mathrm{Tmp}} \Delta \mathbf{h}_{\mathrm{Tmp}} \quad (12)$$

where $\mathbf{Q}_{\mathrm{Tmp}}$ is the nodal transverse flow rate going across the mid-plane element, $\Delta\mathbf{h}_{\mathrm{Tmp}}$ is the nodal head jump across the mid-plane element, and $\mathbf{K}_{\mathrm{Tmp}}$ is the mid-plane element stiffness matrix for the transversal flow:

$$\mathbf{K}_{\mathrm{Tmp}} = \int_L k_t \mathbf{N}\mathbf{N}^{\mathrm{T}} dx \quad (13)$$

3.2.3. Combined Formulation with Double-Nodes

The whole formulation has been developed for the virtual element situated within the interface mid-plane, and now it must be extended to the nodes belonging to the actual interface element. In order to achieve such a goal, we are going to postulate two main hypothesis concerning h_{mp} and Δh_{Tmp}:

$$h_{mp} = \frac{h^- + h^+}{2} \quad (14)$$

$$\Delta h_{Tmp} = h^- - h^+ \quad (15)$$

These last expressions are also true for the element nodes. Therefore, we get the respective expression in terms of the nodal total head $\mathbf{h}^{\mathrm{el}} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix}^{\mathrm{T}}$:

$$\mathbf{h}_{\mathrm{mp}} = \begin{bmatrix} h_{mp_1} \\ h_{mp_2} \end{bmatrix} = \begin{bmatrix} \dfrac{h_1 + h_3}{2} \\ \dfrac{h_2 + h_4}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \mathbf{I} & \mathbf{I} \end{bmatrix} \mathbf{h}^{\mathrm{el}} \quad (16)$$

$$\Delta\mathbf{h}_{\mathrm{Tmp}} = \begin{bmatrix} \Delta h_{Tmp_1} \\ \Delta h_{Tmp_2} \end{bmatrix} = \begin{bmatrix} h_1 - h_3 \\ h_2 - h_4 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{I} \end{bmatrix} \mathbf{h}^{\mathrm{el}} \quad (17)$$

which substituted into equations (8) and (12) give:

$$\Delta\mathbf{Q}_{\mathrm{Lmp}} = \frac{1}{2}\mathbf{K}_{\mathrm{Lmp}} \begin{bmatrix} \mathbf{I} & \mathbf{I} \end{bmatrix} \mathbf{h}^{\mathrm{el}} \quad (18)$$

$$\mathbf{Q}_{\mathrm{Tmp}} = \mathbf{K}_{\mathrm{Tmp}} \begin{bmatrix} \mathbf{I} & -\mathbf{I} \end{bmatrix} \mathbf{h}^{\mathrm{el}} \quad (19)$$

In order to achieve a unique stiffness matrix for the actual interface element we need to couple both longitudinal and normal flows. To tackle this last step, we assume that the leak-off supplies both flows, so that the incoming fluid flux from the boundaries is divided into two parts that feed the longitudinal flow, by means of ΔQ_L^- and ΔQ_L^+, and the normal flow separately. This condition is specified in Fig. 4, which represents the flow rate equilibrium and distribution at a mid-plane node.

This assumption is actually dual to its counterpart on potentials (14), and could have been obtained purely mathematically by invoking reciprocity or applying the Principle of Virtual Work on the system potential-discharge. In this sense we could say that this is the "conjugate" assumption to (14) necessary to obtain symmetric conductivity matrices.

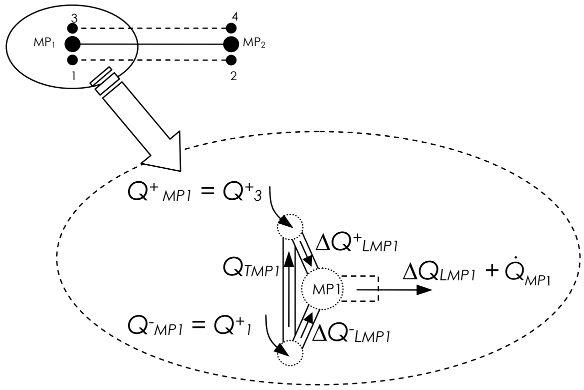

Fig. 4. Distribution of leak-off flow rate between longitudinal and transversal mid-plane flow rates.

According to the diagram represented in Fig. 4:

$$Q^-_{mp_i} = Q_{Tmp_i} + \Delta Q^-_{Lmp_i} \quad (20)$$

$$Q^+_{mp_i} + Q_{Tmp_i} = \Delta Q^+_{Lmp_i} \quad (21)$$

$$\Delta Q^-_{Lmp_i} + \Delta Q^+_{Lmp_i} = \Delta Q_{Lmp_i} + \dot{Q}_{mp_i} \quad (22)$$

If we combine these very last equations we get an analogous expression for the longitudinal flow to that of equation (6):

$$Q^-_{mp_i} + Q^+_{mp_i} = \Delta Q_{Lmp_i} + \dot{Q}_{mp_i} \quad (23)$$

If we now assume as hypothesis $\Delta Q^+_{Lmp_i} = \Delta Q^-_{Lmp_i}$, we reach an expression that relates the transverse flow to the leak-off flow rates:

$$Q_{Tmp_i} = \frac{1}{2}\left(Q^-_{mp_i} - Q^+_{mp_i}\right) \quad (24)$$

Combination of equations (23) and (24) lead up to:

$$Q^+_{mp_i} = \frac{1}{2}\Delta Q_{Lmp_i} - Q_{Tmp_i} + \frac{1}{2}\dot{Q}_{mp_i} \quad (25)$$

$$Q^-_{mp_i} = \frac{1}{2}\Delta Q_{Lmp_i} + Q_{Tmp_i} + \frac{1}{2}\dot{Q}_{mp_i} \quad (26)$$

It is reasonable to do the following assumption:

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} Q^-_{mp_1} \\ Q^-_{mp_2} \end{bmatrix} \text{ and } \begin{bmatrix} Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} Q^+_{mp_1} \\ Q^+_{mp_2} \end{bmatrix} \quad (27)$$

Therefore, combination of equations (18), (19), (25), (26) and (27) lead to the expression we were searching for:

$$\mathbf{Q}_{\text{int}}{}^{\text{el}} = \mathbf{K}^{\text{el}}\mathbf{h}^{\text{el}} + \dot{\mathbf{Q}}^{\text{el}} \quad (28)$$

$$\mathbf{K}^{\text{el}} = \begin{bmatrix} \frac{1}{4}\mathbf{K}_{\text{Lmp}} + \mathbf{K}_{\text{Tmp}} & \frac{1}{4}\mathbf{K}_{\text{Lmp}} - \mathbf{K}_{\text{Tmp}} \\ \frac{1}{4}\mathbf{K}_{\text{Lmp}} - \mathbf{K}_{\text{Tmp}} & \frac{1}{4}\mathbf{K}_{\text{Lmp}} + \mathbf{K}_{\text{Tmp}} \end{bmatrix} \quad (29)$$

$$\dot{\mathbf{Q}}^{\text{el}} = \frac{1}{2}\begin{bmatrix} \dot{\mathbf{Q}}_{\text{mp}} \\ \dot{\mathbf{Q}}_{\text{mp}} \end{bmatrix} \quad (30)$$

4. COUPLING TERMS/MECHANISMS

The hydromechanical problem is solved using a staggered strategy, which makes use of exactly the same FE mesh for both the hydraulic and the mechanical problems, and where an iterative procedure leads to the solution of the coupling.

The methodology starts by solving the hydraulic problem, which gives the nodal total head distribution or, equivalently, the nodal pressure distribution if we make use of the formula:

$$p_f = \gamma_f \left(h - z\right) \quad (31)$$

where γ_f is the specific fluid weight, and z is the nodal elevation head. This fluid pressure distribution is used as input when solving the mechanical problem. Resolution of the mechanical problem gives as output the nodal displacement distribution, which allows us to calculate the normal relative displacement between the two boundaries defining the discontinuity, i.e. the joint aperture, which will influence the value of the terms $\mathbf{K}^{\text{el}}$ and $\dot{\mathbf{Q}}^{\text{el}}$. Resolution of the hydraulic problem with the aperture influence results in a new nodal pressure distribution, so that an iterative procedure is established until a tolerance is satisfied. Fig. 5 summarizes the scheme followed for the staggered strategy related to the transient problem, as for the steady-state case the same procedure would apply just by considering $\dot{\mathbf{Q}}^{\text{el}} = 0$.

The relative displacement vector between the two boundaries defining the interface element is, in terms of the local displacements e_n and e_s:

$$\mathbf{e} = \begin{bmatrix} e_s \\ e_n \end{bmatrix} = \begin{bmatrix} e_s^+ - e_s^- \\ e_n^+ - e_n^- \end{bmatrix} \quad (32)$$

According to the FE formulation, the relative displacement vector at the interface element can be expressed in terms of the global nodal displacement vector ($\mathbf{d}$) as $\mathbf{e} = \mathbf{BRd}$, where

$$\mathbf{B} = \begin{bmatrix} -N_1 & 0 & -N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & -N_1 & 0 & -N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \quad (33)$$

$$\mathbf{R} = diag\left\{\mathbf{r} \quad \mathbf{r} \quad \mathbf{r} \quad \mathbf{r}\right\} \; ; \; \mathbf{r} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \quad (34)$$

We are actually interested just in the normal component of the relative displacement, i.e. e_n, which provides the joint width $w = e_n$.

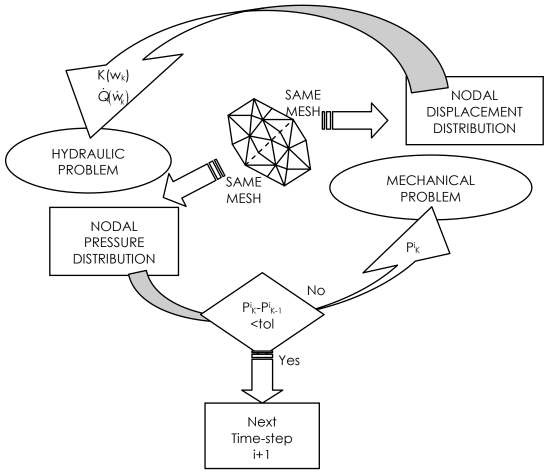

Fig. 5. Staggered procedure scheme

Once we know how to calculate the aperture along the discontinuity, we are ready to calculate the coupling terms. We have already said that there are two of them when studying the hydraulic behavior of an interface element:

a) the capacity term $\dot{\mathbf{Q}}^{\text{el}}$ defined in equation (30), and dependent upon the discontinuity width through the term $\dot{\mathbf{Q}}_{\text{mp}} = \dot{\mathbf{Q}}_{\text{mp}}\left(\dot{w}\right)$ defined in equation (9).
b) the stiffness matrix $\mathbf{K}^{\text{el}}$ defined in equation (29), and dependent upon the joint aperture through the longitudinal hydraulic conductivity, $k_l = k_l\left(w\right)$. If we idealize flow along the discontinuity as being flow between a couple of smooth parallel plates and we assume the flow to be laminar, we reach the following expression for k_l :

$$k_l = \frac{g}{12\nu} w^3 \quad (35)$$

which is usually referred to as the "cubic law" [13], and where ν is the fluid kinematic viscosity and g is the gravity acceleration.

Although the main objective of this article is the study of the hydromechanical coupling within an interface element, it seems appropriate to devote some lines to the coupled behaviour within the continuum medium. The staggered procedure described before also applies for the continuum medium, and the only difference is that the hydraulic analysis within a continuum element is dependent upon the mechanical trough the element volume strain. There exist both a capacity term and a stiffness matrix as in the interface analysis, but now they are dependent upon the continuum porosity, which may be related to the volume strain. The link between the permeability and the porosity may be achieved by using the well known Karman-Cozeny expression [14].

5. CONCLUDING REMARKS

- A new formulation based on the FEM using zero-thickness interface elements and staggered strategy is being developed to solve numerically the coupled hydro-mechanical problem, with application to hydraulic fracture and other processes with mechanical-diffusion coupling through interfaces.
- It is based on a double-node interface formulation for both mechanical and diffusion analysis, and it incorporates transversal flow trough the discontinuity.
- The new interface flow model has several advantages in front of the existing formulations. It is a simpler model than those using triple nodes, with fewer degrees of freedom, and it uses the same nodes for both the hydraulic and mechanical analysis. On the other hand, it is a model that, although being a double-nodded interface, does not lead to indetermination thanks to the introduction of a transverse flow governed by the transversal hydraulic conductivity k_t.
- The same finite element mesh may be used for mechanical and flow analysis, with the obvious subsequent advantages.

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