OPTIMUM REINFORCEMENT FOR DUCTILE RESPONSE OF LRC BEAMS USING A SIMPLE GA

R. C. Yu and G. Ruiz

ETSI de C. y P., Universidad de Castilla-La Mancha,
Avda. Camilo José Cela s/n, 13071 Ciudad Real, Spain

Abstract. The objective of this work is to apply an optimization technique based on the use of a Genetic Algorithm to evaluate the minimum reinforcement in lightly reinforced concrete (LRC) beams for given material properties, geometry and loading conditions. It adopts the *effective slip length* model, which reproduces numerically the mechanical response of a given reinforced beam, as the tool that generates fitness values needed by the GAs procedure. The speed and accuracy of the model compensate with the large amount of calculations required by GAs. The methodology is demonstrated through the searching of minimum ratio for a reinforced prismatic beam loaded at three points at various conditions.

Resumen. Esta comunicación aplica una técnica de optimización basada en un algoritmo genético (GA) para hallar la armadura mínima de una viga de hormigón armado dadas las propiedades de los materiales, la geometría y las condiciones de apoyo. Utilizamos el modelo de la *longitud efectiva de anclaje*, el cual reproduce numéricamente la respuesta mecánica de un aviga armada dada, como la herramienta que genera los valores de ajuste necesarios para los procesos internos del GA. La velocidad y precisión del modelo compensa la gran cantidad de cálculos que el GA tiene que realizar. Validamos la metodología realizando varios ejemplos de búsqueda de la armadura mínima para una viga prismática que resiste flexión en tres puntos.

1 INTRODUCTION

In a reinforced concrete design, in order to prevent instantaneous catastrophic failure without warning, the capacity of a section or a structure as a whole to undergo a reasonable amount of plastic deformation without significant loss of strength during its collapse is preferred. This means minimum reinforcement requirements are necessary to provide a *ductile response*, ensure adequate beforehand knowledge of an impending failure at overloads, and prevent excessive crack widths at service load (in the case of strongly reinforced members).

Various researchers have dedicated themselves to the issue of minimum reinforcement from different viewpoints [1–5]. In all cases, empirical formulas were proposed for practicing engineers. Nevertheless, when new conditions do not fall within the application range of those formulas, the traditional trial-and-error would be the only way to proceed. With these concerns in mind, we would like adopt Genetic Algorithms (GAs), which have been proved effective when "guessing" is needed for decision making, to establish a systematic way, or a *black box* for the searching of the minimum reinforcement.

The basic underlying principle of GAs is that of the Darwinian evolutionary principle of natural selection, wherein the fittest members of a species survive and are favored to produce offspring. First introduced by Holland [6], GAs have since been extensively used to solve optimization problems where conventional methods are either inapplicable or inefficient, ranging from optimal design of stacking sequence of laminate composites [7–11], shape and structural optimization, [12–14], to parameter identification [15, 16], and so forth. But they nevertheless tend to be computationally expensive, since a set of samples with certain degree of variety is needed to start and evolve the searching process. If the fitness value has to be fed by even a moderate finite element method calculation, the amount of calculations needed would render the usage unrealistic. There are two avenues to sidestep this difficulty. One is to use a fast method for computing the fitness value of each member; the other is to avoid the necessity of computing the fitness value for all the members. The method of Pichler, Lackner and Mang [16] belongs to the second category. They incorporated GAs with the trained Artificial Neural Network (ANN) to provide an estimate of optimal solutions, which finally are going to be assessed by an additional FE analysis. The methodology so indicated seem to pave a way for GAs to be used in other large scale finite element analysis. Nevertheless, to start with, we adopt here the first methodology, i.e., using a fast numerical method to feed the objective function for the GAs. Actually, a simple GA (no overlapping of searching samples) is chosen to perform the optimization process.

GAs have been previously adopted by Coello and his coworkers [17, 18] for minimum reinforcement but in a sense of reducing the costs of concrete, steel, etc.; Rafiq [19] applied GAs to optimal design and detailing of reinforced concrete biaxial columns in identifying the optimal reinforcement bar sizes and bar detailing arrangements, but neither of them adopted ductility as the design objective nor is the concept of fracture mechanics referred to.

In this work, we follow the simple and fast numerical method of Ruiz, Elices and Planas [3, 20], the *effective-slip-length model* incorporated with the *smeared tip method*. The model solves the cohesive cracking propagation process through superposition of elastic cases and represents the effect of bond and re-bar by a pair of concentrated forces acting inside the bulk material. A direct application of this work is determining the three point bending behavior of a prismatic beam with a reinforcement layer when a cohesive crack crosses its middle section. Nevertheless, the procedure established is valid for any specimen and any cementitious material reinforced with a single layer of fibers, and it could also be extended to multiple layers. The load-deflection response so obtained depicts several characteristic load points, such as the first and the second peak load, the ultimate yielding load. The ductility is defined by comparing the first peak load and the ultimate yielding load, the closeness of these two loads as well as the current reinforcement ratio are reflected in the GAs' objective function for guiding future search. The input parameters for GAs are chosen based on previous analysis of Ruiz, Elices and Planas [3, 20], whereas new parameters can be incorporated if necessary.

The rest of the paper is structured as follows. Section 2 explains the cohesive model adopted, which is the mechanic model for the cracking behavior of a LRC beam. Section 3 gives an overview of the Genetic algorithm, the description of the numerical model, the objective function as well as the suitable design variables. The numerical examples are accounted for in Section 4. The potential applications and future work are summarized in Section 5.

2 THE MECHANICAL MODEL

In this section we briefly summarize the model of Ruiz, Elices and Planas which describes the propagation of a cohesive crack crossing a reinforcement layer [3, 20]. For a LRC beam loaded at three points, the model assumes several hypotheses regarding three main aspects of the problem: (1) the failure mechanism: only one cohesive crack progresses at the central cross section of the beam (this in fact defines light reinforcement), the cohesive crack is governed by a general cohesive law; (2) matrix and reinforcement behavior : the cohesive behavior of the matrix outside the fracture zone is linear elastic, the reinforcement is considered elastic-perfectly plastic; (3) the bond relation: the shear stress transferred between reinforcement and matrix is a function of the relative slip between both materials and can be replaced by a pair of concentrated forces acting inside the matrix.

Following those assumptions, the cohesive nature can be modeled through the *smeared crack tip* method, while the reinforcement-to-matrix interaction can be dealt with by the *effective slip-length model*. The idea of the *smeared crack tip* method is to consider a nonlinear cohesive crack at the middle cross-section as a superposition of a series of stress-free cracks, whose tips are *smeared* along the propagation path; each of these stress-free cracks is analyzed as a linear elastic case and then summed together to obtain the non-linear state of the cohesive crack [21]. The *effective slip-length model* incorporates the effect of the reinforcement by means of internal stresses, which allows considering the steel-concrete interaction to be located within the concrete, a concentrated force acting at the center of gravity of the shear stress distribution, is adopted to avoid introducing the width of the reinforcement as a further variable.

The solution to the actual numerical problem is sought easily through a triangular system in a computationally inexpensive way (see [20] for details). The model was successfully used to describe the tests on micro-concrete, during which all parameters were determined through independent tests. Five dimensionless parameters were identified to govern the behavior of LRC beams: the Hillerborg's brittleness number, the steel ratio, the relative yield strength of steel, the bond strength and the reinforcement cover thickness. In dimensionless terms they are defined as

$$\beta_H = \frac{D}{l_{ch}}, \quad \rho = A_r/A_c, \tag{1}$$

$$\eta = \sqrt{\frac{E_r}{E_c}\frac{\tau_c}{f_t}\frac{pl_{ch}}{A_r}}, \tag{2}$$

$$\gamma = \frac{c}{D}, \quad f_y^* = f_y/f_t, \tag{3}$$

where D is the beam depth, $l_{ch} = E_c G_F / f_t^2$ is the characteristic length, G_F, the specific fracture energy, f_t, the tensile strength, of concrete; while A represents the cross-section area, E the elastic modulus, with the lower index $_r$ representing re-bar, $_c$ standing for concrete. Additionally, τ_c represents the interface bond strength, p the perimeter of the re-bar; c stands for the cover thickness, and f_y the yield strength of the reinforcement.

We also list here the dimensionless load and the dimensionless loading displacement which are going to be used later on:

$$P^* = \frac{3Pl}{2BD^2 f_t}, \quad \delta^* = \delta\frac{f_t}{G_F}, \tag{4}$$

where B is the beam width, l is the beam span, which is set to be $4D$ in the following calculations; P and δ are the load and the displacement under the loading point, respectively.

Using this model, for a given geometry (the beam depth, width, span, the re-bar cover thickness etc.), material and interface properties (concrete, re-bar and the bond in between), the peak load and the final yielding load can be obtained and compared to define the beam response is brittle or ductile. Having ductility as our objective, a specific design (the combination of a specific geometry and material properties) is therefore rejected or accepted.

3 THE GENETIC ALGORITHMS

As aforementioned, GAs are searching algorithms based on the mechanics of natural selection and genetics. Within this context, the members of certain species may be regarded as candidate solutions to a problem under investigation; they are ranked according to how well they satisfy a certain criterion, and the fittest members are most favored to combine among themselves to form the next generation, which then replace the preceding one. Since fitter members tend to produce even fitter offspring, which represent better solutions to the original problem. After some generations of evolution,

Variable	$\rho(\%)$	γ	f_y^*	η
Range	0.01-0.65	0.01-0.65	10.-266.	1.-129.
Default	0.2	0.1	100.	42.

Table 1: Variational range and default values for each parameter.

the newest members would most probably contain the optimum, or at least, the near-optimum solution.

To use a GA, a solution to the problem has to be presented as a genome (or chromosome) -often a binary array-, which is actually a combination of the set of related parameters encoded as binary numbers. The GA then creates a population of solutions and applies genetic operators such as reproduction, crossover and mutation to evolve the solutions in order to find the best one(s). If the standard definition and implementation of the genetic operators were adopted, the two most important aspects of using a GA are: (1) the genetic representation of the parameters, whose range will determine the possible variety of the ultimate solution; (2) the objective function whose value defines the fitness of a solution.

In principle, all parameters need to be encoded in order that a thorough search of optimization solution is carried out. Based on previous investigations of Ruiz, Elices and Planas [3, 20], the five dimensionless parameters described in Section 2 are the most influential to the mechanical response of the beam. Therefore we choose those parameters as input design variables for GAs, the corresponding variational values and default values, except the brittleness number, which is set as 1.0, are listed in Table 1. The default values correspond to a case that has been well studied in [3,20], and these are the values adopted for a single-parameter study later on. The variational range for each variable are chosen to reflect the ordinary engineering practice for LRC beams.

In order to define the objective or fitness function for the GAs,we need to have in mind that the design options that give brittle response would have to be rejected, while the ones that yield ductile response should obtain a positive fitness value (this is required by GAs). Although all the aforementioned five dimensionless parameters play a role in the beam response, only the effect of the reinforcement ratio ρ over ductility is obviously monotonic, we will hence explicitly include it in the fitness function. By defining an intermediate parameter $t = P_y^*/P_f^* - 1$, with P_f^* being the dimensionless peak load, P_y^* the normalized ultimate load, a brittle beam response would correspond to a negative t, the ductile behavior would give $t \geq 0$, while the closeness to $t = 0$ would be the near-optimum reinforcement we are looking for. With the help of an exponential function, which is not exclusive, we can write the objective function as follows:

$$f(\rho, \eta, \gamma, f_y^*, \beta_H) = exp(-\rho)exp(-t), \quad t \geq 0. \qquad (5)$$

When t is negative, the fitness value is set to be zero. To obtain P_y^*, the limit case of complete yielding of the re-bar and the re-bar being the sole support of loading is considered, a simple equilibrium condition of the beam gives $P_y^* = 6\rho f_y^*(1-\gamma)$; while P_f^* can be obtained using the mechanical model described in Section 2. Nevertheless, for a typical load-deflection curve obtained through the mechanical model described above, two load peaks could be observed, see Fig.1; even though the first peak represents a physical local maximum, the second peak signals the yielding of the re-bar, due to the elastic-perfectly plastic constitutive behavior assumed (see [3] for details). Thus we identify the first peak as P_f^*, then for each specific parameter distribution, we can obtain a fitness value for that design using Eqn. 5, which would guide GAs for searching new solutions. The design procedure is summarized in Table 2.

1. Define the variational range for all the parameters and set the default values.
2. Encode all the parameters to a binary array.
3. Randomly generate N binary arrays.
4. Decode the binary arrays into N random initial design options.
5. Input the N design options into the mechanical model and compute the fitness value for each design.
6. The N fitness values are given to GAs for generating new binary arrays.
7. Go back to 4, until a certain number of generations are reached.
8. Choose the design option that has the maximum fitness value.

Table 2: The design procedure using GAs.

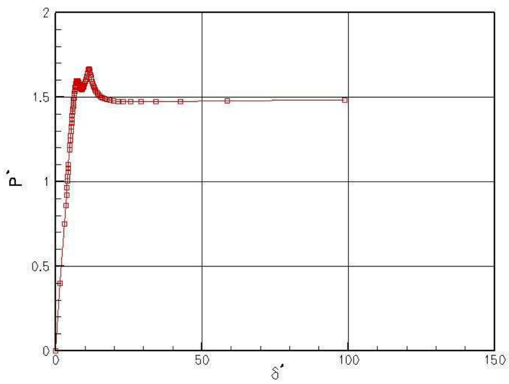
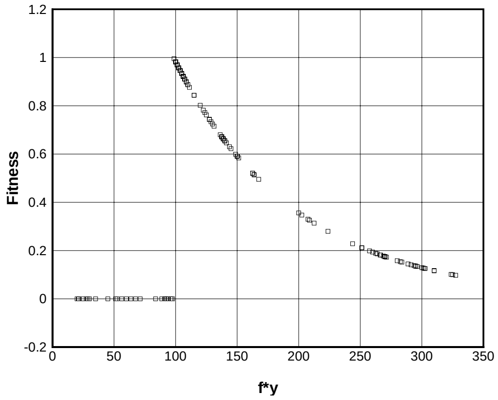


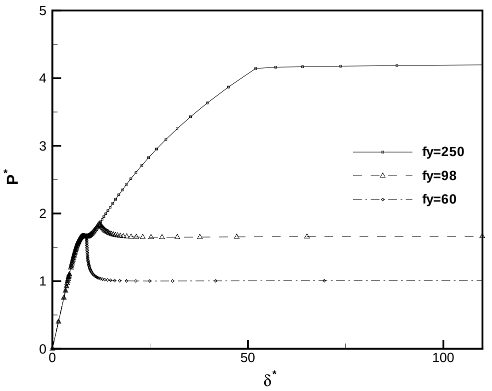
Figure 1: A typical dimensionless load-displacement curve.

4 NUMERICAL EXAMPLES

During previous works, Ruiz, Elices and Planas [3, 20] have done numerous runs using the mechanical model described above and derived a closed-form expression for minimum reinforcement of LRC beams. One of the objectives of this work is precisely to leave this searching procedure to GAs. By feeding GAs the design variables, we wish to obtain the optimized design solution. In order to validate the methodology described above, we first present some results of one-parameter variation and verify the solution by analyzing directly the mechanical response (the load-displacement curve in this case), then a 2-variable solution is discussed to show that the methodology can be easily extended to multiple variables.

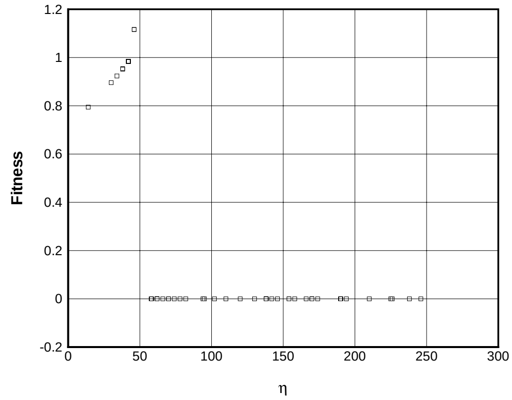


(a)

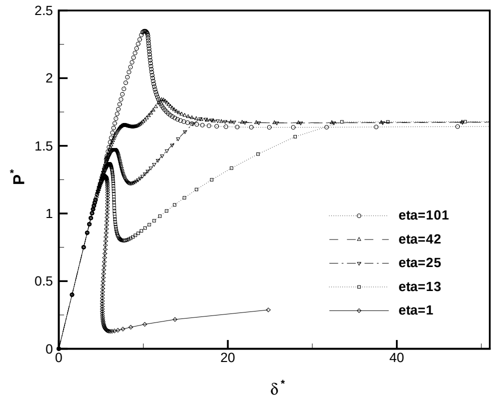


(b)

Figure 2: The distribution of the objective function with respect to (a) the dimensionless yield strength and (b) the load-displacement curves for f_y^* = 60, 98 and 250.



(a)



(b)

Figure 3: The distribution of the objective function with respect to (a) the dimensionless bond strength and (b) the load-displacement curves for η = 1, 13, 25, 42 and 101.

4.1 When the dimensionless yield strength f_y^* is the only variable

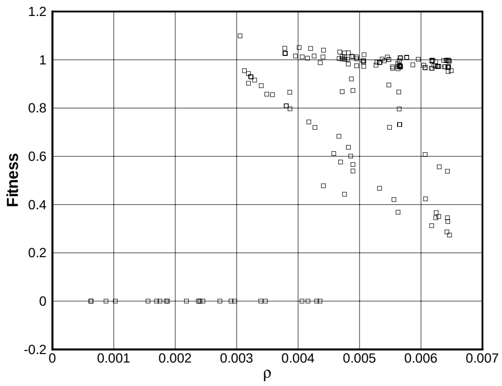
The influence of f_y^* the response of the beam, as illustrated Fig.(2), is similar to that of the reinforcement ratio, i.e., when all other parameters are fixed, the re-bar needs a minimum strength f_{ymin}^* in order to ensure a ductile response; once f_{ymin}^* is reached, increasing f_y^* would overly reinforce the beam. Another interesting observation is that, since f_y^* is defined as the ratio between the steel yield strength and the concrete tensile strength, if higher strength concrete were assumed, higher steel yield strength would be required, provided that the rest conditions remain the same, as shown in Figure 2.

4.2 When the dimensionless bond η is the only variable

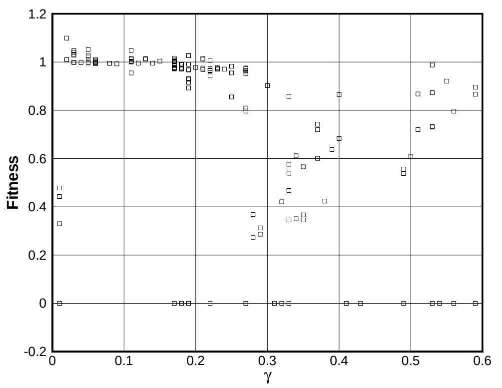
Figure (3a) shows the distribution of the objective function with respect to the dimensionless adherence η. It is noted that in the beginning when η increases, the fitness value also increases, but after a certain adherence value, the fitness value dropped to zero, which means the beam response ceases to be ductile. An interesting observation is that there exists a minimum and maximum value of η so that the beams response would remain ductile. A weaker bond strength than the minimum would not activate the reinforcement (the case of no bonding is equivalent to plain concrete), whereas a stronger bond strength than the maximum would signal a false hyper-strength which is not supported by the re-bar yielding strength.

4.3 When the dimensionless cover thickness γ and the reinforcement ratio ρ are the only variables

In Fig. 4, we present the fitness variation with respect to both the reinforcement ratio and the normalized cover thickness, with the other variables taking their default values. In spite



(a)



(b)

Figure 4: The distribution of the objective function with respect to (a) the reinforcement ratio (b) the normalized cover thickness.

of the apparent randomness distribution, a maximum fitness value of 1.1 can be identified for $\rho = 0.305$ and $\gamma = 0.02$, which corresponds to a ratio of 1.09 between the ultimate and the first peak load. Although the value of 0.02 for γ is not realistic, since a minimum reinforcement cover thickness was not taken into consideration in the calculations, it is nevertheless consistent with the idea that smaller γ takes better advantage of the reinforcement.

5 SUMMARY AND CONCLUSIONS

We have incorporated a robust optimization technique (the Simple GA) with a fast numerical method (*the effective slip-length model*) dealing with cohesive crack propagation in a LRC beam. Our objective was to make the procedure of searching for minimum reinforcement systematic and at the same time supported by sound mathematical tools. The selected results presented for both single and two-parameter variations, not only validate the methodology, but also show promising aspects for dealing more complicated cases of minimum reinforcement design for LRC beams.

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