Buckling of Weakened X-Bracing System

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RESUMEN

Este trabajo presenta un nuevo estudio de los efectos de elementos debilitados en sistema de *X-bracing* sobre la carga crítica de pandeo. El método propuesto combina las técnicas de la mecánica de fractura con la teoría de estabilidad elástica. La debilidad se ha modelado como muelles de rotación basados en la energía liberada. El modelo propuesto predice adecuadamente las cargas críticas de pandeo y es una fácil herramienta para evaluar la influencia del daño sobre el factor de longitud efectiva, en sistema de *X-bracing*.

ABSTRACT

A novel study for the effect of weakened members in X-bracing system on the critical buckling load is presented. The proposed method is combining the fracture mechanics techniques and the theory of elastic stability. The weakness is modelled as rotational springs based on energy release rate. The proposed model presents an accurate prediction for the critical buckling loads and an easy way to evaluate the damage influence on the effective length factor, in X-bracing system.

KEY WORDS: X-Bracing, Effective Length Factor, Fracture mechanics.

1. INTRODUCTION

The elastic stability of cross-bracing members is of a practical importance in the structural design of struts and ties, which are widely used in the construction of steel structures.
A cross-brace consists of two slender members in the form of X. They are either simply or rigidly attached at their end connections to the framework, and are pinned or rigidly connected at their point of intersection. The system is free to deflect in or out the plane of the framework. In X-braced structure, the tension brace provides lateral support to the compressed one. Even if both are in compression, significant lateral elastic support is achieved to the compression brace with higher axial load by the lower loaded one.
The problem was firstly analyzed considering the compression member laterally supported by an elastic support, using generalized three moment equations [1, 2]. Several studies have focused attention on the behaviour of X-bracing systems. The problem was formulated using differential equations assuming equal section properties and equal lengths [3]. This problem was similarly addressed providing some experimental results [4]. The formulation of the stability criteria using the energy approach for the general case where the tension and the compression braces are different in length, properties and boundary conditions was presented [5].
The problem of cracked individual beam-columns has been studied extensively [6, 7 and 8], as the cracks reduce the flexural rigidity of structural elements. Also the effect of internal hinges on buckling analysis of rectangular plates was studied [9].
The aim of the present paper is to study the effect of cracks on the behaviour of X-bracing systems. The effect of reduced rigidity of the tension brace or low loaded compression brace on the critical buckling load of the main compression brace is analyzed.
This work presents a novel model for the evaluation of the load carrying capacity, the deflection as well as the lateral stiffness of X-bracing system having a single-edge crack in one of its members. The proposed model is based on the theory of elastic stability together with the well-known relationship between the compliance and the stress intensity factor of a cracked beam [10]. Using the conditions of compatibility and equilibrium between the two components of X-bracing system, the critical buckling load can be calculated. The method is applicable where the tension and compression brace forces are different in value and sign. The results have a good agreement with well established methods.

2. BEAM-COLUMN EQUATIONS

In this section, the expression for the deflection of a hinged-hinged beam-column under the effect of both axial force at its ends and a lateral load at the mid point of its length is obtained using the differential equation of deflection.

The assumptions employed in the study are:

- All members are of a rectangular cross section and the stresses in these members remain within the limit of proportionality.
- The connection between X-bracing members conveys lateral force only.

Consider the X-bracing system shown in Figure 1 subjected to an axial compression force *P* and an axial tensile force *Q*. The out of plane deflection, the first, and second derivatives represented by y, dy/dz, and d^2y/dz^2 respectively. Also, the terms *E*, *I*, *L* are the modulus of elasticity, moment of inertia and length of members.

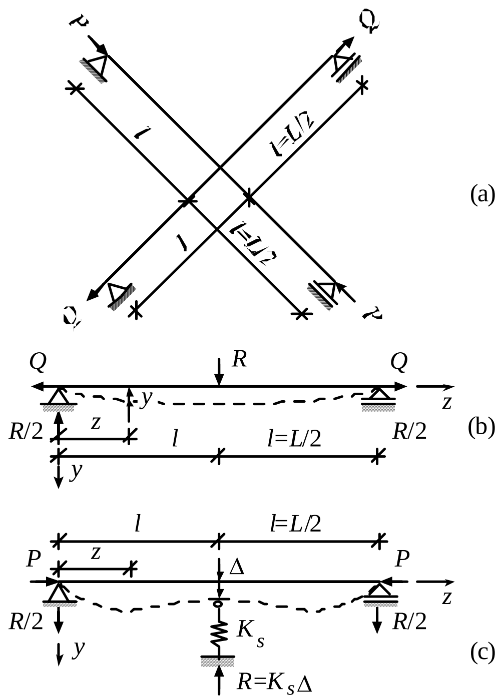

Figure1. X-Bracing System.

2.1. Cracked beam-column subjected to a tensile force.

Figure 2 shows a single-edge cracked beam-column with hinged ends. The beam-column has a rectangular cross-section of a width *B* and a thickness *W*. The beam-column sustained a tensile force *Q* at its ends and a lateral force *R* at mid span.

The beam-column is divided into three segments [6]; the central segment of a length l^*, which contains a crack of a length *a*, and two outer segments without cracks each of a length *l*.

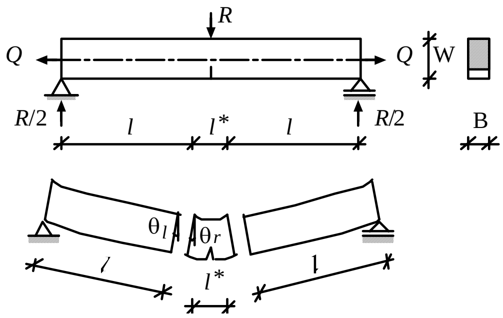

Figure 2. Cracked Beam-column under Tension.

The differential equation of the elastic curve of the beam-column in this case is given by:

$$\frac{d^2y}{dz^2} - \lambda^2 y = -\frac{R}{2EI} z \qquad (1)$$

where: $\lambda^2 = -\dfrac{Q}{EI}$

The general solution is given by:

$$y = A\sinh\lambda z + B\cosh\lambda z + \frac{R}{2EI\lambda^2} z \qquad (2)$$

Using the continuity conditions of the moment and the slope of the beam-column at the intersection between the central and the outer segments, and $y = 0$ at $z = 0$, the constants *A* and *B* can be calculate. The end conditions lead to *B*=0. Therefore, the deflection, the slope and the moment are given by

$$y = A\sinh\lambda z + \frac{R}{2EI\lambda^2} z \qquad (3)$$

$$\frac{dy}{dz} = \lambda A\cosh\lambda z + \frac{R}{2EI\lambda^2} \qquad (4)$$

$$M = -EI\lambda^2 A\sinh\lambda l \qquad (5)$$

At the intersection of the outer and central segments (z=$L/2$=l where l^* tends to zero). The moment and slope, *M* and θ, of the outer segment are related to the lateral deflection. The moment and slope M^* and θ^*, of the central segment are related to each other by the beam compliance of the cracked central segment [10].

Thus the bending moment at both ends of central segment and the relative rotation θ^*, between these two ends, must follow the relation

$$M^* = 2k\theta^* \qquad (6)$$

Where *k*, is the rotational spring constant.

Combining equations (4), (5) and (6) leads to the determination of constant *A*

$$A = \frac{-R/Q}{\left(EI\lambda^2/k\right)\sinh\lambda l + 2\lambda\cosh\lambda l} \qquad (7)$$

Noticing the fact that $\theta^{*}=2\theta=2dy/dz$. The beam-column deflection can be represented as

$$y=\frac{R}{Q}\left[\frac{z}{2}-\frac{\sinh\lambda z}{\left(EI\lambda^{2}/k\right)\sinh\lambda l+2\lambda\cosh\lambda l}\right] \tag{8}$$

If one assumes that the length of the central segment is zero, i.e. $l^{*}=0$, and the relative stiffness $\beta = EI/2kl$ then, the deflection of mid point of the beam-column Δ can be calculated as

$$\Delta=\frac{Rl}{2Q}\left[\frac{\left(\beta\lambda^{2}l^{2}-1\right)\sinh\lambda l+\lambda l\cosh\lambda l}{\beta\lambda^{2}l^{2}\sinh\lambda l+\lambda l\cosh\lambda l}\right] \tag{9}$$

2.2. Cracked beam-column subjected to compressive force.

Likewise, Figure 3 shows a single-edge cracked beam-column with the characteristics similar to the previous case but subjected to a compressive force Q.

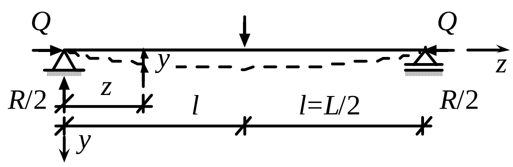

Figure 3. Cracked Beam-column under Compression.

In this case the differential equation of the elastic curve is given by:

$$\frac{d^{2}y}{dz^{2}}+\lambda^{2}y=-\frac{R}{2EI}z \tag{10}$$

Considering the continuity of the moment and the slope at the intersection of the central and the outer segments, and y=0 at z=0. The general solution is:

$$y= A\sin\lambda z+B\cos\lambda z-\frac{R}{2EI\lambda^{2}}z \tag{11}$$

The end conditions lead to $B = 0$. And the deflection, the slope and the moment are given by:

$$y= A\sin\lambda z-\frac{R}{2EI\lambda^{2}}z \tag{12}$$

$$\frac{dy}{dz}=\lambda A\cos\lambda z-\frac{R}{2EI\lambda^{2}} \tag{13}$$

$$M=-EI\lambda^{2}A\sin\lambda z \tag{14}$$

The normal stress distribution, σ_Q and σ_M, over the uncracked section, due to an axial compressive force Q and a moment M is shown in Figure 4.

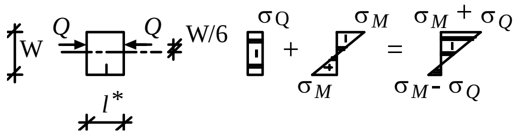

Figure 4. Normal Stress over Uncracked Section .

The crack will affect the column stiffness only if $(\sigma_M+\sigma_Q)>0$. Resolving the concentric compressive force and the applied moment into an eccentric compressive force located at $W/6$ and a moment M^{*}, the opening up moment is $(QW/6+M^{*})$, where M^{*} is given by:

$$M^{*}=2k\theta \tag{15}$$

Thus, as the moment M^{*} applied to the crack gradually, the crack opened up, and the stiffness is reduced. Equating the bending moment and slopes at both central and outer segments leads to:

$$EI\lambda^{2}A\sin\lambda l=2k\lambda\left(A\cos\lambda l-\frac{R}{2EI\lambda^{3}}\right)+\frac{QW}{6} \tag{16}$$

Solving this equation, the value of the constant A is determined in the form:

$$A=\frac{\left(Rl/2Q\right)-\left(W\beta\lambda^{2}l^{2}/6\right)}{\lambda l\cos\lambda l-\beta\lambda^{2}l^{2}\sin\lambda l} \tag{17}$$

The beam-column deflection in this case is:

$$y=\left[\frac{\left(Rl/2Q\right)-\left(W\beta\lambda^{2}l^{2}/6\right)}{\lambda l\cos\lambda l-\beta\lambda^{2}l^{2}\sin\lambda l}\right]\sin\lambda z-\left(Rl/2Q\right) \tag{18}$$

Neglecting the term $(WB\lambda^{2}l^{2}/6)\sin\lambda l$, the deflection at mid span of the beam-column Δ is defined as:

$$\Delta=\frac{\left(Rl/2Q\right)\sin\lambda l}{\lambda l\cos\lambda l-\beta\lambda^{2}l^{2}\sin\lambda l}-\left(Rl/2Q\right) \tag{19}$$

2.3. Intact beam-column subjected to a compressive force

Referring to figure 1-c, the main compression brace can be modelled as a beam-column supported by a spring of a reaction R. The differential equation of this beam-column is defined by:

$$\frac{d^{2}y}{dz^{2}}+\mu^{2}\,y=\frac{R}{2EI}z \tag{20}$$

where: $\mu^{2}=-\frac{P}{EI}$

And its solution is:

$$y=A\sin\mu z+B\cos\mu z+\frac{R}{2EI\mu^{2}}z \tag{21}$$

The boundary conditions in this case are $y = 0$ at $z = 0$, $dy/dz = 0$ at $z = l$ and the deflection at $z = l$ has the value Δ. The solution is defined by:

$$y = \left(\Delta - \frac{Rl}{2EI\mu^2}\right)\frac{\sin \mu z}{\sin \mu l} + \frac{R}{2EI\mu^2} z \qquad (22)$$

And the deflection at mid point is equal to:

$$\Delta = \frac{R}{2EI\mu^3}\left(\mu l - \tan \mu l\right) \qquad (23)$$

2.4. Compliance of cracked beam-column

The lateral deflection of cracked beam-column increases as a result of the increased compliance C. The relation between compliance and stress intensity factor K_m is given by [11]:

$$\frac{\left(1-\nu^2\right)K_m^2}{E} = \frac{M^2}{2B}\frac{dC}{da} \qquad (24)$$

And the value of the spring constant k is defined by:

$$k = 1 \Big/ \frac{1}{E}\left[\frac{l^*}{I} + \frac{72\left(1-\nu^2\right)}{BW^2}F\left(a/W\right)\right] \qquad (25)$$

Where:

$$\begin{aligned} F\left(a/W\right) = {} & 1.98\left(a/W\right)^2 - 3.277\left(a/W\right)^3 + 14.43\left(a/W\right)^4 \\ & - 31.26\left(a/W\right)^5 + 63.56\left(a/W\right)^6 - 103.36\left(a/W\right)^7 \\ & + 147.52\left(a/W\right)^8 - 127.69\left(a/W\right)^9 + 61.5\left(a/W\right)^{10} \end{aligned}$$

Substituting for $l^*=0$, we get

$$k = \frac{EBW^2}{72\left(1-\nu^2\right)F\left(a/W\right)} \qquad (26)$$

3. STABILITY CRITERIA FOR THE WEAKENED X-BRACING SYSTEM

At this stage, it is now possible to determine the critical buckling load of the main compression member of X-bracing system. The criteria is based on equating the deflection calculated at mid span of cracked tension or lower loaded compression brace to the deflection of the main compression intact brace. Then by solving the characteristic equation of μl, the load carrying capacity P_{cr} can be founded.
The characteristic equation for the case of weakened tension brace is:

$$\left(\mu l - \tan \mu l\right) = \frac{1}{m^3}\left[\frac{\left(\beta\mu^2 m^2 l^2 - 1\right)\sinh \mu ml + \mu ml \cosh \mu ml}{\beta\mu ml \sinh \mu ml + \cosh \mu ml}\right] \qquad (27)$$

While the characteristic equation for the case of weakened compression brace is:

$$\left(\mu l - \tan \mu l\right) = \frac{-1}{m^3}\left[\mu ml - \frac{\sin \mu ml}{\cos \mu ml - \beta\mu ml \sin \mu ml}\right] \qquad (28)$$

where: $m = \dfrac{\lambda}{\mu}$

4. APPLICTION OF THE MODEL AND RESULTS

The verification of the proposed model is realized by comparing the results obtained from equations (27) and (28) in case of uncracked section, β=0, with those of [5], as shown in Figure 5. In which, the effective length factor is given by:

$$K = \sqrt{\frac{\left(1-\gamma\right)}{2}} \geq 0.5$$

Where $\gamma=m^2=Q/P$

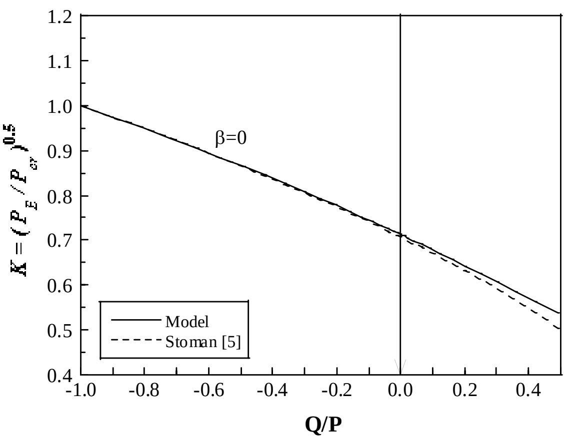

Figure 5. Comparison of Effective Length Factor.

Also, the predicted effective buckling length factor (when β=0) for the value of Q/P=0.5 using the proposed method is 0.54, where the value determined by [3] for the same value was 0.53.
Figure 6 shows the values of the effective length factor for different ratios of both brace tensile and compressive loads, Q/P, and the weakness represented by the relative stiffness factor, β.

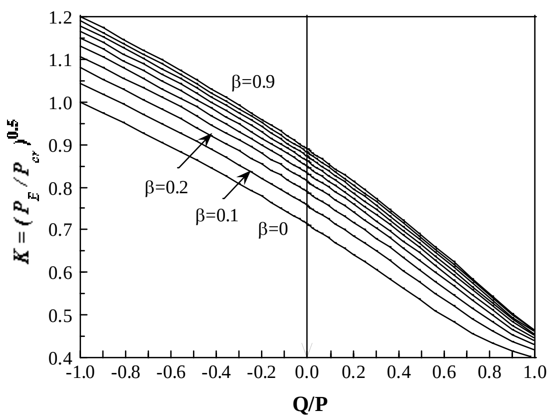

Figure 6. Effective Length Factor for Weakened X-Bracing.

5. CONCLUSIONS

A novel model for the stability of weakened X-bracing system has been proposed and verified. The model is formulated using the differential equations for cracked and uncracked beam-column for different cases of axial force. The results for case of β=0 were confirmed by other analytical methods. The calculated values for the effective buckling length factor of weakened X-bracing system show that a considerable reduction for the load carrying capacity is achieved with increasing of the β value. The model is accurate, simple and drives a set of design aid curves.

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