CRACK PROPAGATION IN PLANE STRAIN UNDER VARIABLE AMPLITUDE LOADING

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ABSTRACT

Crack propagation simulation began with developing of finite element method; the analyses were conducted to obtain a basic understanding of the crack growth and closure processes. Today structural and materials engineers develop structures and materials properties using this technique. In this paper procedures to determine the crack opening and closure by finite elements analyses in plane strain will be presented. The objective of this paper is also provide a review of retardation models under variable spectrum loading considering plane strain constraint as well as their correlation with experimental data.

KEY WORDS: Crack Propagation Simulation, Finite Element Method, Variable Amplitude Loading, Plane strain.

1. INTRODUCTION

The major links between fatigue and fracture mechanics were done by Christensen [1] and Elber [2]. The crack closure concept put crack propagation theories on a firm foundation and allowed the development of practical life prediction for variable and constant amplitude loading, as experienced by modern day commercial aircrafts, wind turbine components and automotive applications. Analytical models were developed to predict crack growth and crack closure processes like the Dugdale model [3].

2. REVIEW OF CRACK PROPAGATION MODELS

In Newman [4] a review is presented of a chronological development of fracture mechanics and crack propagation simulation. The list of some of the finite-element and finite-difference analyses [5-14] is given in Table 1; the vast majority of these analyses were conducted using two-dimensional analyses under either plane-stress or plane-strain conditions. Since the mid-1980s, only a few three-dimensional finite element analyses have been conducted basically Neman [10] and Chermahini [11]. Today have others authors that work with tridimensional models in special in plane strain like Roychowdhury & Dodds [15] and Josefson et al. [16].

Table 1. Finite element models of fatigue crack growth and closure

Two-dimensional cracks	Three-dimensional cracks
Newman and Armen [5]	Newman et al. [10]
Ohji, Ogura and Ohkubo [6]	Chermahini [11]
Blom and Holm [7]	Chermahini et al. [12,13]
Fleck and Newman [8-9]	Dawicke et al. [14]

Newman and Armen [5] and Ohji *et al.* [6] were the first to conduct two-dimensional, finite-element analyses of the crack-closure process. Their results under plane-stress conditions were in quantitative agreement with the experimental results of Elber [17] and showed that crack-opening stresses were a function of R ratio ($S_{min}//S_{max}$) and stress level (S_{max}/σ_0). Blom and Holm [7] and Fleck and Newman [8-9] studied crack-growth and closure under plane-strain conditions and found that cracks did close but the crack-opening levels were much lower than those under plane-stress conditions.

Matos & Norwell [18] present a literature review of the phenomenon of plasticity-induced fatigue crack closure under plane stress and plane strain conditions and mention that there are controversial topics concerning the mechanics of crack propagation. In general there is no consensus in the scientific community. Table 1 shows the Chronological crack advance scheme.

Table 1: Chronological crack advance scheme

Year	Author	Node Release Scheme	Constraint	Target	Element Type
1985	Blom and Holm [7]	Maximum load	PStress; PStrain	COP and CCL	Triangle linear
1986	Fleck and Newman [8]	Maximum load	PStress; PStrain	COP	Triangle linear
2000	Wei and James [19]	Maximum load	PStress; PStrain	COP and CCL	Triangle linear
2002	Pommier [20]	Minimum Load	PStrain	COP and CCL	Quadrilateral linear
2003	Solanski [21]	Maximum load	PStress; PStrain	COP and CCL by COEL	Quadrilateral linear
2004	Solanski et al. [22]	Maximum load	PStress; PStrain	COP and CCL by COEL	Quadrilateral linear
2004	Zhao et al. [23]	Maximum load	PStrain	COP and CCL by CME	Quadrilateral linear
2005	Gonzalez-Herrera and Zapatero [24]	Maximum load	PStress; PStrain	COP and CCL by DME	Quadrilateral linear

PStress- plane stress; *PStrain*- plane strain; *COP*- crack opening; *CCL*- crack closing; *COEL*- crack opening and closing by contact element; *CME*- crack opening and closing by compliance method; *DME*- crack opening and closure by displacement method

Numerical methods, such as the finite element method are often used to simulate plasticity-induced fatigue crack closure in 2D geometries under plane strain [2–6]. The phenomenon of plasticity-induced fatigue crack closure under plane strain conditions is one of the most controversial topics concerning the mechanics of crack propagation. No general consensus exists among in the scientific community concerning the physical mechanism for crack closure under plane strain conditions. One of the problems is in the way to prepare the mesh and the procedure used in crack propagation. With the three-dimensional models it becomes necessary to use normal contact approach to node release; in plane stress spring is normally used to help the crack propagation, using contact issues for crack propagation and considering nonlinear analysis it will result in a big result file and will spend a considerable time processing to end the simulation.

Fleck [2] According to his work the source of discontinuous closure appears to be a residual wedge of material on the crack flanks, located just ahead of the initial position of the crack tip. He suggested that closure involves only a few elements relatively distant from the current crack tip and the closure levels decay steadily as the crack grows beyond its initial length.

More recently Wei and James [19] reported that after growing a plane strain fatigue crack for a few cycles, there is no contact in the region immediately behind the crack tip and the contact pressure along the crack faces is discontinuous. Zao et al. [17] modelled a CT specimen under plane stress and plane strain. They did not observe plasticity-induced crack closure under plane strain during steady state crack growth under cyclic tension, although they found significant levels of closure under plane stress. Chermahini et al. [11] present the crack propagation in 3D and in plane strain to determine the crack opening level, the crack-opening stress was found to vary through the thickness for a middle-crack tension specimen.

On the specimen surface and in the mid-plane the crack-opening stress levels tend to be two-dimensional solutions for plane stress and plane strain conditions, respectively. Figure 1 shows the model used by Chermahini et al. [12]. In Figure 2 it is possible to see the finite element model used for crack propagation. The model was prepared in layer of elements, considering the lower element in the reverse plastic zone computed by Irwin equation and then will increasing the size of hexahedron element until arriving in region that the results will not affect the stress level in the crack propagation area. Spring was used spring for node release cycle after cycle like Newman [10].

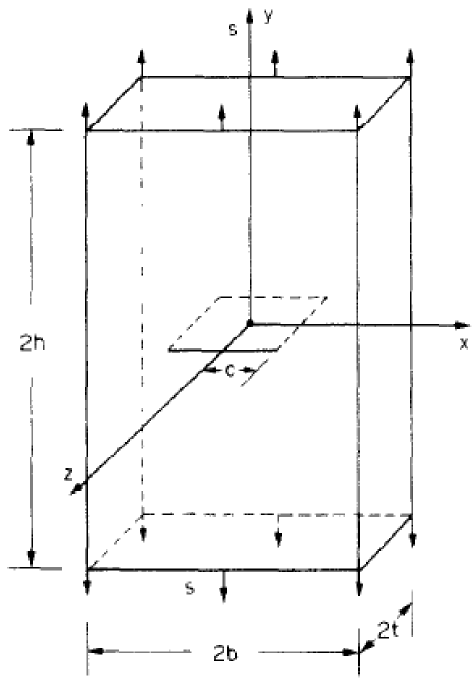

Figure 1 Middle-Crack Tension Specimen Subjected to uniform stress [11]

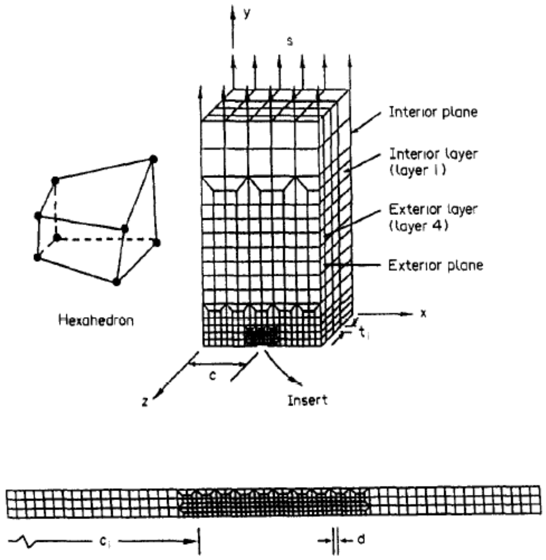

Figure 2 Crack Propagation Model Quarter of Middle Tension [11]

Solanki et al. [22] present a review of crack propagation in plane stress and plane. Figure 3 shows a M(T) specimen modeled with an externally induced T -stress to observe the subsequent change in closure levels under plane-strain. A T -stress was induced by applying tractions parallel to the crack in addition to the conventional tractions perpendicular to the crack. They found a significant drop in the closure level as the T -stress was varied from compressive to tensile as shown in Figure 3. Figure 4 also shows the difference of result in node release at minimum and maximum load compared by Solanski et al. [22].

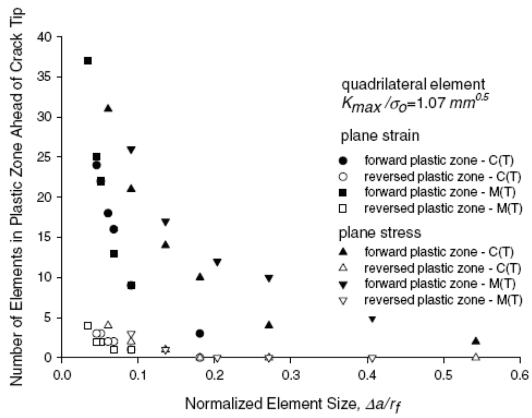

Figure 3 Variations in Crack Tip Plastic Zone Size with Mesh [22]

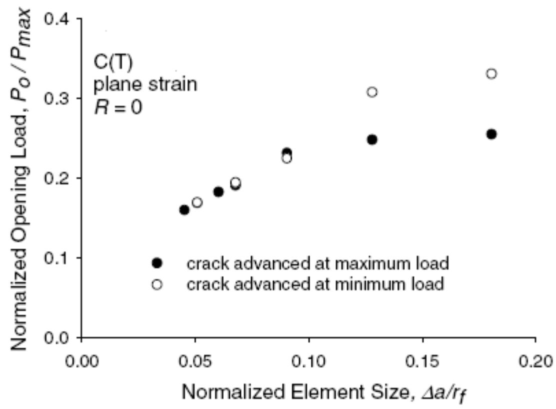

Figure 4 Comparison of Crack opening values based on crack advance scheme [22].

Wu and Ellyin [25] have used a truss element together with pairs of contact elements and the element death option for crack propagation simulation.This technique used in plane stress and plane strain models is usual in commercial finite element codes. The element death option was incorporated to remove truss elements. With their approach, a node can be released any time during a load cycle irrespective of the magnitude of the deformation caused by the release of the node. Consequently, fewer problems with convergence were encountered and also several nodes could be released simultaneously if desired.

3. FATIGUE CRACK GROWTH UNDER VARIABLE AMPLITUDE LOADING

Schijve [26] was one of the first to study the process of fatigue crack growth (macrocracks) under variable-amplitude loading was extensively investigated in numerous experimental programs.
The promising application of the ΔK-concept to predictions of fatigue crack growth under constant-amplitude loading was drastically upset by the first experiments with overloads in constant-amplitude tests carried out around 1960 [26, 27].

Three overloads depicted in Figure 5 induced highly retarded crack growth.

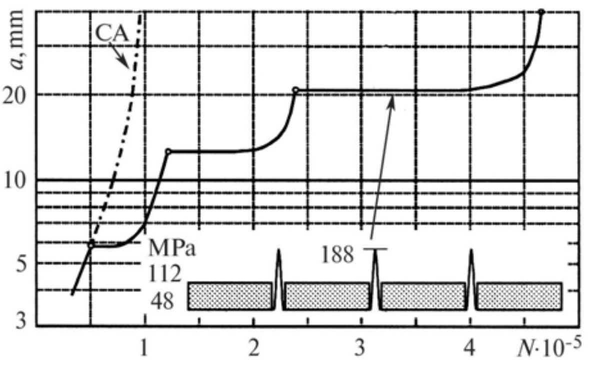

Figure 5 Effects of Overloads in Sheets Specimens

Originally, this was attributed to compressive residual stresses in the crack-tip plastic zone of an overload. Later, it was related to crack closure in the plastic zone leading to smaller effective stress ranges. Actually, there is no contradiction between these two explanations because both are associated with plastic deformation in the crack-tip zone. The pronounced effect of high loads on fatigue crack growth stimulated a lot of research, both experimental investigations and analytic studies.

The significance of crack-tip plasticity was easily recognized and an obvious assumption was made that the size of the plastic zone must be important for crack-growth retardations.

Since the size of the plastic zone depends on the stressed state (plane strain or plane stress, or intermediate situations), the retardation of the growth of a through crack after an overload must depend on the thickness of the material and its yield stress. This observation was amply confirmed by experimental results, starting in the mid-70s (see, e.g., [28, 29]). More detailed observations also indicate that the maximum retardation of the crack-growth rate after an overload does not occur immediately but after a certain period of time required for the penetration of the crack tip into the plastic zone of the overload (the so-called delayed crack-growth retardation).

The first analytic models of crack growth under variable-amplitude loading were based on the sizes of plastic zones.

The Willenborg [30] and Wheeler [31] models are two notable examples published in the early 70s. They are now considered to be rather primitive. The second generation of crack-growth reduction models for variable amplitude loading was based on the effect of plasticity-induced crack closure. The crack extension Δai in a cycle i is a function of ΔKeff in this cycle. Thus, we can write

$$a = a_0 + \sum \Delta a_i, \quad \text{where} \quad \Delta a_i = \left(\frac{da}{dN}\right)_i = f(\Delta K_{\mathrm{eff},i}). \tag{1}$$

The quantity ΔKeff depends on the applied σmax, i and predicted σop, i stresses. The crack-opening level of stresses σop must be predicted by a crack-growth model taking into account the effect of plastic deformation left in the wake of the crack as a residue of the previous loading cycles. As an illustration, in Figure 6, we present a sample of a variable-amplitude loading history with variable values of σop. The quantity Δσeff is determined by using a relation similar to relation (1).

Figure 6 Variation of the Crack-Opening Stress

Corbly & Packman [32] describe some aspects of the retardation phenomenon. Despite the recent increase in research into retardation effects in crack propagation, there are many aspects of load interaction phenomena that lack adequate explanations. Three aspects of the retardation phenomena that are generally agreed upon are presented below.

1. Retardation increases with higher values of peak loading σ_{peak} for constant values of lower stress levels [33,34].

2. The number of cycles at the lower stress level required to return to the non-retarded crack growth rate is a function of ΔK_{peak}, ΔK_{lower}, R_{peak}, R_{lower}, and number of peak cycles [35].

3. Increased percentage delay effects of peak loading, given a percent overload, are greater at higher baseline stress intensity factors [36].

Ljustell, P. & Nilsson, F. [37] perform an investigation of crack growth in notched specimens with part through cracks, subjected to variable amplitude block loading.

A related goal investigated is if closure corrected LEFM methods could predict crack growth at relatively high loads. In order to compare the closure levels estimation as outlined above with numerical predictions finite element simulations of plasticity induced crack closure was performed with the commercial code, ABAQUS v.6.3 [38]. A two-dimensional finite element model, plane strain, with four-node quadrilateral elements of the NT specimen was used for the analyses. Figure 7 shows the good correlation with numerical and experimental results.

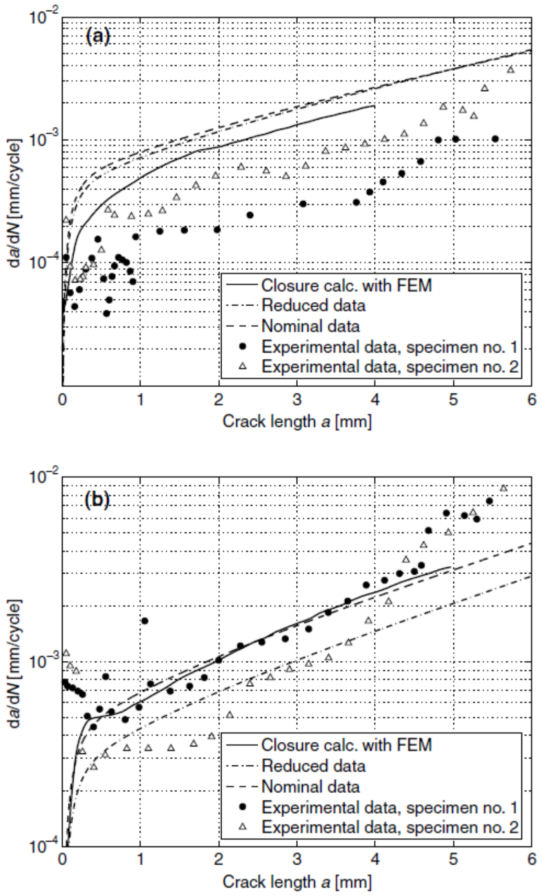

Figure 7 Results from the NT-experiments marked with dots and predictions as lines: (a) and (b) are cycle type 1 and 2 respectively [37]

4. CONCLUSION

The paper presented a review of some procedures used to simulate crack propagation under variable amplitude loading. In the literature there are many different techniques used and of course much discussion also in terms of applicability of these techniques in a general case. Normally the authors work with particular cases and few load blocks history to avoid complexities in the edition of signal for example. There is much work to do in plane strain due the models in order to run requests on non usual informatics facilities.

The good agreement is that the time processing must be lower than a real crack propagation test.

This is one of the challenges the researchers have to solve as well as simulating with good quality of results the threshold area.

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