HYBRID FORMULATION SOLUTION FOR STRESS ANALYSIS OF CURVED PIPES WITH WELDED JOINTS UNDER BENDING

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ABSTRACT

A curved thin pipe subjected to an in-plane uniform bending moment is studied. A mixed formulation for the stress analysis of curved pipes is proposed and applied to an elbow where the stress field is evaluated, represented by a continuous analytical function obtained by unknown functions combined with trigonometric expansions. Then, these results are used as input data for the calculation of the stress intensity factor of a part-through crack existing in the plane of the flange-pipe junction. The SIF is evaluated for the defect in the flange-pipe interface of a Sandvik® 3R12 stainless steel elbow, considering different pipe thickness and crack geometries at an in-plane bending load.
A comparison with results published in the literature for a welded structure with some kind of flaws due to structural defects, lack of material adhesion or porosities is presented, validating this approach.

1. INTRODUCTION

In the present work a mixed formulation solution where unknown functions, ovalization and warping, are used together with Fourier trigonometric expansions [1,2,3] is presented. These shape factors are used in the evaluation of SIF parameters for a generalized crack existing in a doubly curved surface shell.
In previous works published by Viswanatha *et al* [4] stress intensity factors in elbows subjected to uniform in-plane bending where a part-through crack along the longitudinal axis at the crown pipe were obtained using a finite element method to perform the analysis. Also Oliveira *et al* [5] studied the stress intensity factors of surface elliptical cracks existing in pipe elbows using high order 8-node shell elements. The methodology presented here is a new, alternative approach, based on a coupled procedure, where in a first step, the nominal stress field of the curved pipe loaded as mentioned is evaluated; after, a database containing the parameters necessary to evaluate the stress intensity factor of part-through cracks existing in shell structures is used for the objective. In the present analysis, a previous investigation performed by Carpinteri *et al* [6] is used. These authors calculated the stress intensity factor of several geometries of elliptical profile part-through cracks existing in cylinders subjected to uniform bending. The cracked pipe structure was modelled with shell type finite elements.
The analysis now presented refers to a toroidal shell that has two principal curvatures, while a cylindrical shell has in fact only one curvature. The approach taken for this purpose has the assumption that the curvature along the longitudinal direction is much larger than the one along the circumferential line. To assess the accuracy of the approach here mentioned, it was also taken a set of reference values of the stress intensity factor calculated by Gonçalves and Castro [7] referred to a part-through crack existing in flat plate where the authors used the Line-Spring model techniques. In this work it´s interesting to note that the so-obtained values of the stress intensity factor for a part-through crack in a flat plate are very similar to the same parameter for a cylindrical shell as in the work of Carpinteri *et al* [6], provided that the cylinder was not considered as a thick shell (*i.e*, the ratio radius/thickness greater than about 20).

2. STRESS ANALYSIS IN A CURVED PIPE UNDER BENDING

In a curved pipe with end flanges, the flange-pipe interface is the main flaw location, as sketched in Fig. 1.

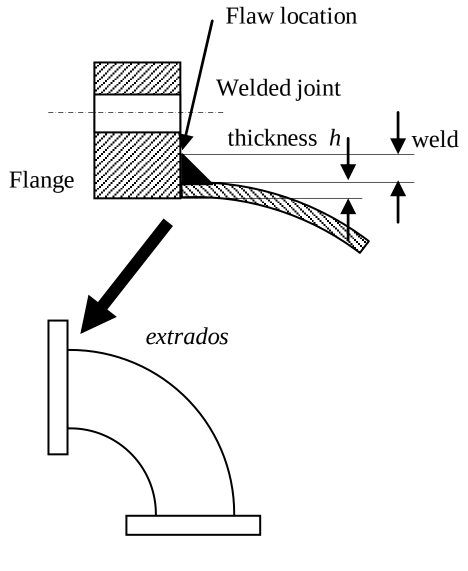

Fig 1- Part-through crack in a welded flange joint of a curved pipe

The purpose of this work is to analyze the structural integrity of a pipe where a crack similar to the one displayed in Figure 1 is detected. The stress intensity factor (SIF) associated with the crack shape is used in the analysis to quantify the remote or nominal stress field that exists in the crack zone. This work follows Carpinteri *et al* [6], where straight pipes that have a part-through crack were subjected to bending loads. Due to the presence of the defect, the pipe integrity is defined by the maximum SIF along the crack profile. The shell considered here is not cylindrical, but considering the pipe curvature radius *R*, much larger than the transverse section radius *r*, this approach is still valid.

The solution is obtained by minimizing the total elastic energy *U* involved in the pipe distortion process with a set of unknown functions:

$$U(\boldsymbol{\sigma},\boldsymbol{u}) = \frac{1}{2}\int_{\Omega} \boldsymbol{\sigma}^{\mathsf{T}} \boldsymbol{C}^{-1} \boldsymbol{\sigma}\, d\Omega + \int_{\Omega} \boldsymbol{\sigma}^{\mathsf{T}} \boldsymbol{u}\, d\Omega \tag{1}$$

C is the elastic pipe properties matrix. The first integral refers to the internal elastic deformation energy, and the second is the deformation work performed by the unknown vector **σ** of the internal stresses against the admissible displacement field ***u***.
An external entity M_0, the bending moment, performs additional work at a bending angle $\delta\alpha$.
The system of external forces performs deformation work (called the external work) upon the displacement functions a_i and b_i. The total energy of the structure *U* is the sum of this external work with the corresponding elastic deformation. This value should be zero if the displacement functions are exact, but for the displacement field approximations used here (Fourier expansions), *U* has a residual value. This residual allows for the optimization of expressions to minimize *U*.
The variation of the total energy *U* is as follows:

$$\delta U = \int_{\Omega} \delta\boldsymbol{\sigma}^{\mathsf{T}} \boldsymbol{C}^{-1} \boldsymbol{\sigma} d\Omega + \int_{\Omega} \delta\boldsymbol{\sigma}^{\mathsf{T}} \boldsymbol{u} d\Omega + \int_{\Omega} \boldsymbol{\sigma}^{\mathsf{T}} \delta\boldsymbol{u} d\Omega \tag{2}$$

where the integral domain Ω is $[0, L]\times[0, 2\pi]$ and *L* represents the length of the transverse section of the center arch. The total variation in *U* is the result of separate contributions from variations in the stress field **σ** and in the displacement ***u***. Equilibrium is achieved by minimizing the function *U* separately for each of the unknowns and a system of differential equations is then obtained [4,5]. Using MAPLE® mathematical solver an analytical solution is achieved and the continuous stress field is evaluated [1,2] for the location in discussion.

3. EXAMPLES
A curved pipe where the shell surface close to the crack zone is approximately cylindrical is considered. The method is applied to 3 examples of curved pipes with cracked rigid end flanges. Figure 2 shows a crack resulting from the lack of adhesion that occurs in the *extrados* curved pipe. The pipe is subjected to a bending moment that results from a prescribed in-plane flange rotation of 0.002 rad, as described in Figure 2.

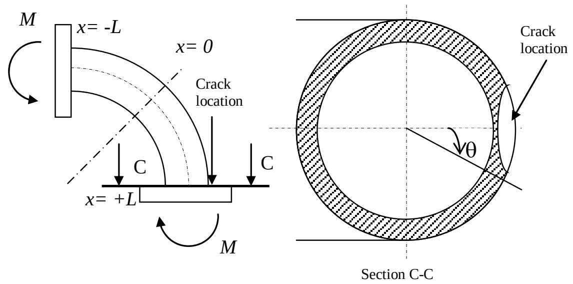

Fig 2- Part-through crack in a welded flange joint in a curved pipe

The maximum bending stress σ_M at the *extrados* position in the flange weld is as shown in Figure 3. The corresponding value for the stress intensity factor in $MPa\times m^{1/2}$ is shown in Table 1. The critical SIF value for the stainless steel used here is $K_{IC}\approx 28\ MPa\times m^{1/2}$ (http://www.smt.sandvik.com/tube/). To assess the solution method discussed here, stress analysis is carried out for several curved pipes with rigid end flanges. The material properties and dimensions are:

EXAMPLE 1
Transverse section radius *r* – 50 *mm*
Wall thickness *h* – 5*mm*
Mean curvature radius *R* – 200 *mm*
Pipe angle α – 90º
Transverse section center arch *L* – 157.0796 *mm* (=*R*α)
Material – stainless steel Sandvik® 3R12l: E=210 GPa $\nu = 0.3$
An in-plane uniform bending moment M_0 loads the pipe, where symmetry conditions were considered. In the calculations, a specified rotation angle of δα=0.002 rad at a flanged end was prescribed.

EXAMPLE 2
With the same material dimensions are:
Transverse section radius *r* – 50 *mm*
Wall thickness *h* – 2*mm*
Mean curvature radius *R* – 200 *mm*

EXAMPLE 3
With the same material dimensions are:
Transverse section radius *r* – 40 *mm*
Wall thickness *h* – 2*mm*
Mean curvature radius *R* – 200 *mm*

The hoop stress at section *x*=*L*, as a consequence of the bending moment, is shown in Figure 3:

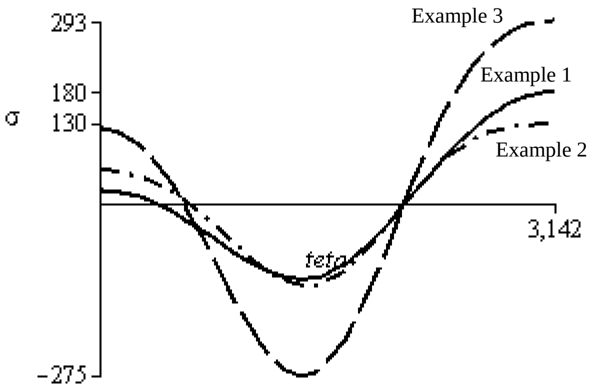

Fig. 3 – Longitudinal membrane stress σ_x at x=L from uniform bending

The following parameters are necessary for evaluating the Stress Intensity Factor (SIF) associated to the crack geometry (Figure 4):

a = maximum crack depth
a_{el} =minor half axis of ellipse (the crack has an elliptical profile)
ξ=a/t (non-dimensional parameter; measured at the largest crack depth)
α= a_{el}/b_{el} (ellipse principal axes ratio)
s=a_{el}/a (offset of ellipse major axis related to the tangent to the external pipe circumference; usually s=1).

An auxiliary coordinate ζ, refers to the position point in the crack profile where the stress intensity factor is measured; coordinate ζ starts at the intersection point of the crack profile with the pipe's external circumference; ζ is used as a normalized ratio ζ/h (see Figures 5)

For example 1
D (pipe section diameter) =100*mm* (r=50*mm*)
t (equivalent thickness = pipe+weld) =10*mm*

For example2
D (pipe section diameter) =100*mm* (r=50*mm*)
t (equivalent thickness = pipe+weld) =5*mm*

For example 3
D (pipe section diameter) =100*mm* (r=40*mm*)
t (equivalent thickness = pipe+weld) =4*mm*

Semi-elliptical crack is respectively a=2*mm* a=3*mm* a=1.6*mm* (a=a_{el}); a/t is 0.6 in example 2 and R/t is 10 or 5 and are compared to the values available in the graphical results of Carpinteri [7]

Material: austenitic stainless steel Sandvik® 3R12

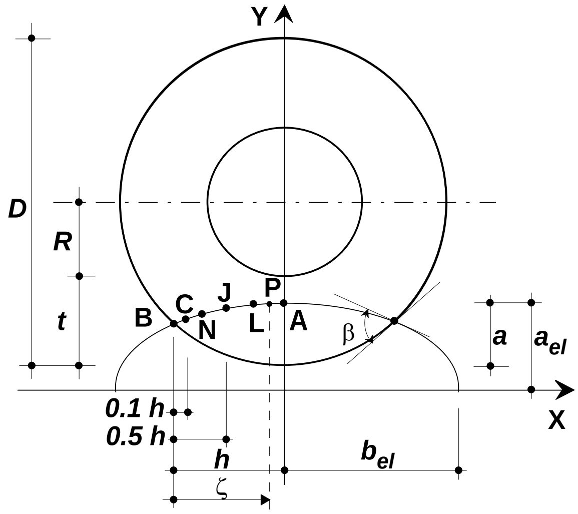

Fig. 4 – Parameterization of a semi-elliptical part-through crack existing in a pipe [7]

Given these parameters, the non-dimensional stress intensity factor, in reference to the nominal value, can be estimated:

$$K_{\mathrm{I}} = \sigma_M \sqrt{\pi \, \mathrm{a}}$$

The case study for example 2 has a=3*mm*; the nominal value for K_I is:

$$K_{\mathrm{I}} = \sigma_M \sqrt{\pi \, \mathrm{a}} = \sigma_M \sqrt{\pi \times 3E-0.3}$$
$$= 0.097 \times \sigma_M \; N \times m^{1/2}$$

Figures 5 show the non-dimensional SIF for a defect in a straight pipe and with the geometric parameters described above. Again, using analysis from Carpinteri [7], the non-dimensional SIF is $K^*_{I,M} \cong 1.08$. The effective value of the stress intensity factor is (Table 1):

$$K_{IM} = K^*_{LM} \times \sigma_M \sqrt{\pi \, \mathrm{a}} = 1.08 \times \sqrt{\pi \, \mathrm{a}} \times \sigma_M$$
$$= 1.08 \times 0.97 \times 130 = 13.6$$

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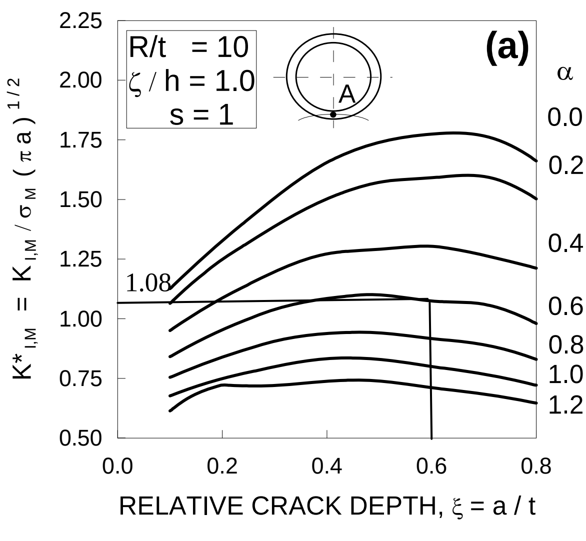

Fig. 5a – Dimensionless Stress Intensity Factor for a part-through crack with geometry parameters from [7]

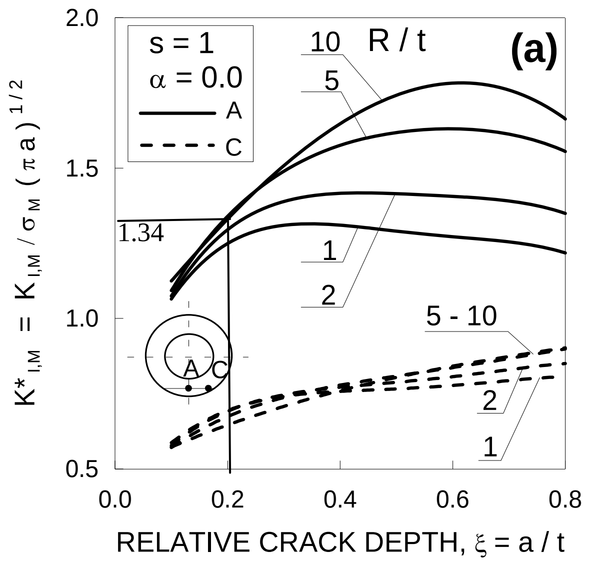

Fig. 5b – Dimensionless Stress Intensity Factor for a part-through crack with geometry parameters from [7]

In example 3 the effective value of the stress intensity factor is (Table 1):

$$K_{IM} = K^{*}_{LM} \times \sigma_M \sqrt{\pi\,\mathrm{a}} = 1.08 \times \sigma_M \sqrt{\pi\,\mathrm{a}}$$
$$= 1.34 \times 0.071 \times 293 = 27.8$$

The longitudinal curvature was assumed to be much smaller than the circular transverse section, and SIF parameter results were obtained using a cylindrical shell.
This method was also validated using a toroidal shell element, assuming flat plate-like properties at the crack zone following the method of Gonçalves and Castro [7]. In these examples, SIFs of semi-elliptical part-through cracks in flat plates subjected to pure tension or bending were studied.

Table.1 – Stress intensity factors for part-through cracks in flat plates under tension

Sample (mm)	Weld (mm)	σ max at θ=π	Crack geometry	SIF
r=50 h=5	weld=5	180MPa	a=.002m	19.1
r=50 h=2	weld=3	130MPa	a=.003m	13.6
r=40 h=2	weld=2	293MPa	a=.0016m	27.8

The method used by Gonçalves [7] to calculate the SIF has the factor *Q*, given by the following equation:

$$Q = 1 + 1.464(a/c)^{1.65}$$

In example 2, *a/c* =0.6, giving *Q* =1.63. The SIF calculated by Gonçalves [7] applied to flat plates is

$$K_{I(plate)} / \sqrt{Q} = 1.3 / \sqrt{1.63} \, @ 1.018$$

The results show that the safety factor is exceeded for the third sample. In the two other examples, the defect does not give a critical threat for pipe integrity, provided that the stress σ_M remains below 280 *MPa*, a value close to the yield stress for the steel alloy.

This value is close to the result obtained by Carpinteri [6] using cylindrical shells. These results express that the method proposed here is suitable for thin shells with relatively large curvatures.

4. CONCLUSIONS

This result shows the method's reliability when applied to a doubly curved shell with a part-through crack, provided that the shell has two distinct principal curvatures, where one is much smaller than the other. As can be shown by example 3 even with a smaller initial crack, because the radius is shorter, the SIF shows that under the same conditions, this is the pipe that breaks reaching the safety factor.

5. AKNOWLEGEMENTS

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