METHODOLOGY FOR IN-SITU STRESS INTENSITY FACTOR DETERMINATION ON CRACKED STRUCTURES BY DIGITAL IMAGE CORRELATION

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ABSTRACT

A methodology for in-situ stress intensity factor (SIF) determinations that can be used for the analysis of cracked structures is shown. The proposed method is demonstrated in the laboratory for the case of a central cracked plate, subjected to uniaxial tension. Three steps are used for the SIF determination. First the strain field around the crack tip is acquired using a digital image correlation based optical technique. Secondly, the stresses are calculated based on the equation from the theory of elasticity. In the third step an over determined system of equations is solved containing the stress field around the crack tip and the stress intensity facto among others.
A comparison of the obtained results was performed with results obtained using the Dual Boundary Element Method (DBEM) together with the J-Integral method for SIF determination. A good agreement can be noticed for both the stress distribution around the crack tip and for the SIF calculated based on these stresses, proving thus the ability to measure the SIF in-situ.

KEYWORDS: Fracture mechanics, in-situ stress intensity factor determination.

1. INTRODUCTION

Modern lightweight structures are often based on damage tolerance design principles, which means that the structure has to withstand the existence of cracks up to the next routine maintenance inspection or up to its defined economical end of life. The maintenance intervals have to be chosen in a way that guarantees a secure usage of the structure, which often leads to high inspection costs.
A better knowledge of the real effect of existing cracks could lead to a better planning of the maintenance operations after a crack has been found in a structure. The possibility to measure the real stress intensity factor (SIF) of a crack in-situ is therefore of high interest.
Since the ability to tolerate a substantial amount of damage is a requirement for modern lightweight structures, it has become increasingly important to develop methodologies to predict failure in fatigue damaged structures. The damage tolerant philosophy must ensure the continued safe operation of structures, which means that a structure is supposed to sustain cracks safely until it is repaired or its service life has ended. Strength assessment of structures is necessary for their in-service inspection, repair, rehabilitation, and health monitoring. The damage tolerance analysis should provide information about the effect of cracks on the strength of structures. Damage tolerance analyses can be performed using linear elastic fracture mechanics (LEFM) concepts where the stress intensity factor is a fundamental parameter. Fracture mechanics in conjunction with crack growth laws, *e.g.* Paris law [1], is widely used to analyze and predict crack growth and fracture behavior of structural components. To study crack growth and to evaluate the remaining life of a certain structural component, rigorous numerical analyses have to be performed to compute SIFs.
Structures can suffer fatigue damage throughout their service life leading to constant changes in geometry. The assumptions made during the design phase are therefore constantly changed, and numerical analyses previously performed no longer accurately show the stress distribution at critical locations. With the ability to monitor the fatigue process in-situ, non-destructive evaluation (NDE) methods are of critical importance for structural integrity evaluation and failure prevention of engineering components in service. The development of experimental techniques to obtain the SIF in real structures is therefore of high interest.

The technique presented in this work can provide a powerful experimental tool to investigate localized inhomogeneous damage and to analyze complex fatigue processes. A better fatigue-life estimation becomes feasible and reduced maintenance costs may be expected as a result.

Digital Image Correlation (DIC) is an optical analysis method that can be used for full field, non-contact 2D and 3D measurement of deformations and strains on the surface of components [2]. Longitudinal and transverse strains with any load, such as tensile, compression, bending and torsion or a combination of different loads can be monitored. The method is based on the correlation of two images acquired before and after deformation. The recorded images are analyzed and compared by a special correlation technique, which allows the determination of the surface displacements with high local resolution. In order to achieve a good correlation, the method uses a speckle pattern applied to the object surface and tracks the gray value pattern in small neighborhoods called subsets during deformation.

One of the advantages of DIC is that no physical sensor has to be installed. This type of measurement system is flexible, since it allows measuring almost any type of deformation in time and space, giving access to information about strain gradients and their variations in time. A disadvantage of this process is that it works better in laboratory under known conditions. Care has to be taken to have a clear view; fumes and similar can create noise and distorted results.

A multipoint overdeterministic method is used for SIF calculation, where experimental data collected from optical images is fitted to Muskhelishvili's equations describing the stress field around the crack tip [3]. The procedure is based on the overdeter-ministic approach, used previously in fracture mechanics for processing photoelastic data in experimental determination of SIFs [4, 5]. The values of the stresses in an unlimited number of points around the crack tip can be used in order to fit a multiterm series expansion of the stress field. A system of equations is obtained in which the coefficients of each term are the unknowns. The number of equations is equal to the number of the considered points while the number of unknowns is equal to the number of terms chosen in the series expansion, which is much lower. This overdeterministic method has the advantage of being able to use an unlimited number of data points, thus minimizing the error.

In order to evaluate the accuracy of the proposed algorithm for SIF determination, the results were compared with the numerical ones obtained using the Dual Boundary Element Method (DBEM).

1.1. The specimen

A pre-cracked MT specimen according to ASTM E647 [6] with a width of 80 mm was selected for this experiment due to its size, which is large enough for a good measurement area and small enough to maintain the fatigue loads relatively low. The specimen thickness was 4 mm and the 10 mm long initial notch was machined by spark erosion (EDM). A drawing of this specimen is shown in Figure 1.

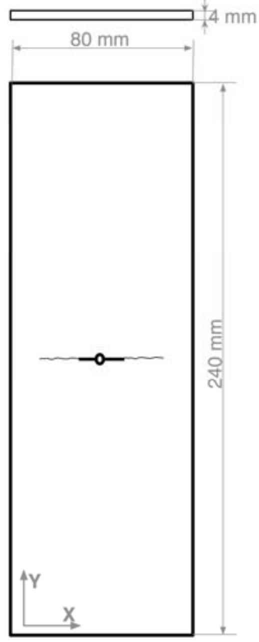

Figure 1. MT Specimen.

One advantage of this type of specimen is the fact that it has a geometry which allows the analysis of symmetric and non-symmetric cracks.

1.2. Fatigue crack growth test

80000 cycles have been performed in order to develop a fatigue crack from the initial notch. The crack half length on the monitored side was a2 = 12.31 mm, measured from the center of the crack. Table 1 shows the parameters used for fatigue loading, and Figure 2 shows a scheme of the studied crack geometry.

Table 1. Fatigue test properties

Property	Value
F_{mean}	11980 N
$F_{amplitude}$	9792 N
R-ratio	0.1
frequency	4 Hz
Number of cycles	80000

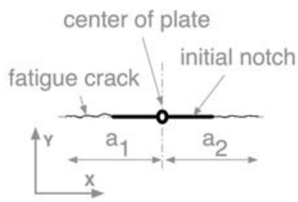

Figure 2. Scheme of the studied crack geometry.

2. Experimental SIF determination

An optical strain and deformation measurement system based on digital image correlation is used to measure the strain distribution around a fatigue crack tip. Based on the obtained measurement results, the stress intensity factor can be calculated by fitting the experimentally obtained results in the analytical stress expression near the crack tip, in the form of a series expansion with different number of terms.

The experiments have been performed in DEMEC/FEUP on a servo-hydraulic MTS 312.31 testing machine using a 250 kN load cell. Ambient temperature was 24ºC during the whole experiment. Figure 3 shows the experimental setup.

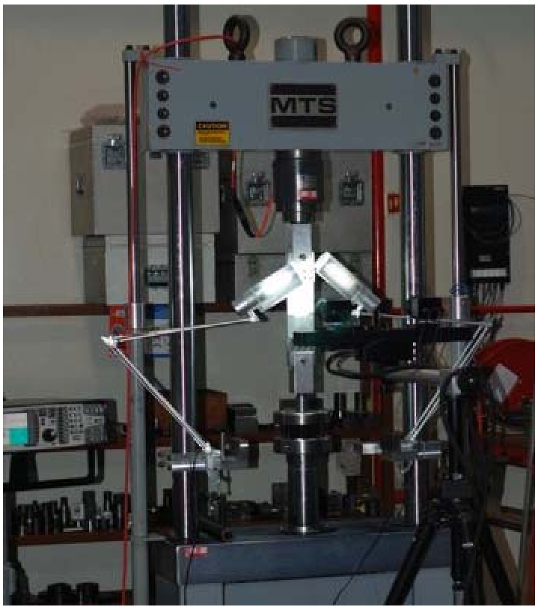

Figure 3. Experimental setup for measurement of the strain around the crack tip.

2.1. Optical strain and deformation measurement system

Only the right crack tip has been measured in this work. The aim was to demonstrate the feasibility of determining the SIF in-situ by optical means. In future works both crack tips in different loading conditions will be monitored.

For the images taken in the unloaded state, a force of only 200 N was considered, in order to allow the specimen not to stay fixed during the measurement which was performed under loadcontrol on the servohydraulic system. The loaded state was measured at the maximum load of the fatigue test, namely F_{max} = 21760 N. The crack extension was measured using a travelling optical microscope attached to a Mitutoyo digital scale with a 1/100 mm resolution.

A portable GOM (Gesellschaft für Optische Messtechnik) ARAMIS 6.0.2 workstation was used for measurement of the strain field around the crack tip. This system is based on digital image correlation, and therefore information about recognizable facets has to be given to the system.

The GOM 2M hardware was used with 50 mm focal length Schneider-Kreuznach lenses with a maximum aperture f = 2.8. Two 18 W white fluorescent lamps at 6400 K were used for illumination of the specimen surface.

The specimen is painted with a stochastic pattern necessary for the digital image correlation operation. A fine black dotted pattern is applied to the white-grounded surface for better contrast. The quality of the results proves to be high when such a prepared surface is used. Figure 4 shows the stochastic pattern applied to the specimen surface, which is used for strain calculation.

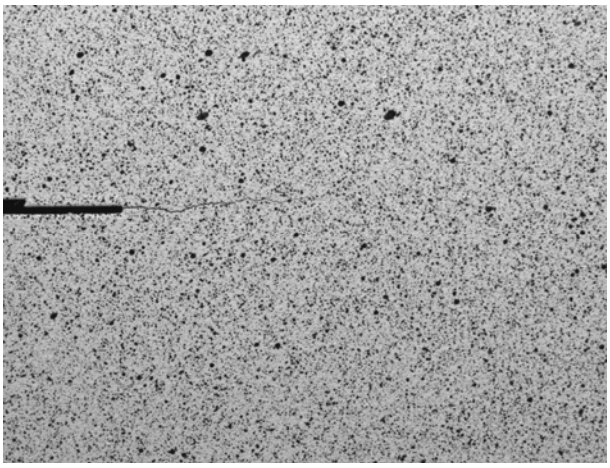

Figure 4. Stochastic pattern applied to the specimen surface for strain calculation; the crack can be recognized due to the maximum load that was applied for this image.

2.1.1. Measurement results

The painted specimen is virtually divided into facets with a size of 7×7 pixels and a facet step of 5 pixels. This means that every 5 pixels a new 7 pixel long facet starts, and overlap between facets is 2 pixels, corresponding to 29% in X and Y directions. Due to the small distance from the lenses to the specimen, an area of around 20×15 mm, corres-ponding to 324×246 facets, is measured. Strain is calculated by measuring the deformation of a facet in relation to its neighbouring facets. In order to obtain a smooth measurement, 19×19 facets where chosen for strain measurement. This is equivalent to a gauge length of about 1.21 mm in traditional measuring instruments. The validity quote was chosen to be 55%, which means that at least 55% of the 19x19 facets have to exist in order to allow the calculation of strain for a certain facet.

Figure 5 shows the obtained strain distribution around the crack tip in the direction perpendicular to the crack plane used for calculation of the SIF.

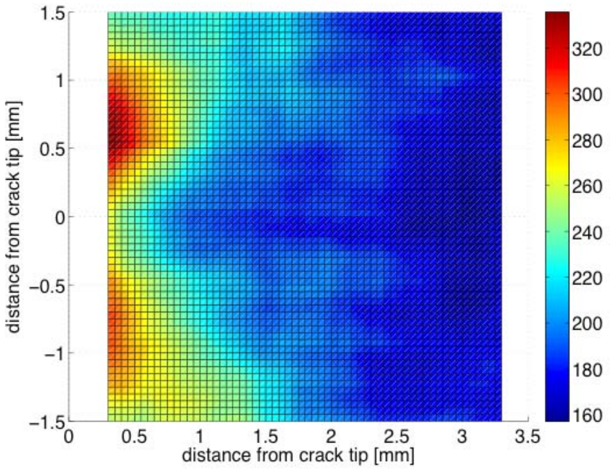

Figure 5. Smoothed strain data in Y direction; εY units are με. The crack tip is located at the centre of coordinates

2.2. Over determined SIF calculation

Knowing the stress field around the crack tip, it is possible to apply an overdeterministic SIF calculation method as was shown in [4] for example.

For the plane stress problem of a homogeneous isotropic solid, in the absence of body forces, the global field equations for the stress components in the vicinity of a straight front crack under mode I conditions can be written as shown by [7]:

$$\sigma_{xx} = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+\sigma_{ox}+$$
$$+\sum_{n=3}^{\infty}\left(A_n\frac{n}{2}\right)r^{\frac{n}{2}-1}\left\{\left[2+(-1)^n+\frac{n}{2}\right]\cos\left(\frac{n}{2}-1\right)\theta+\right.$$
$$\left.-\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\right\},$$
$$\sigma_{yy} = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+$$
$$+\sum_{n=3}^{\infty}\left(A_n\frac{n}{2}\right)r^{\frac{n}{2}-1}\left\{\left[2-(-1)^n-\frac{n}{2}\right]\cos\left(\frac{n}{2}-1\right)\theta+\right.$$
$$\left.+\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\right\},$$
$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}-$$
$$-\sum_{n=3}^{\infty}\left(A_n\frac{n}{2}\right)r^{\frac{n}{2}-1}\left\{\left[(-1)^n+\frac{n}{2}\right]\sin\left(\frac{n}{2}-1\right)\theta+\right.$$
$$\left.+\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta.\right.$$

The origin of the Cartesian (x, y) and polar (r, θ) coordinate systems is defined at the crack tip.

From the equations above, it can be verified that all three exhibit linear dependence with the coefficients K, σ_{0x}, A_3, ... , A_n. This means that a linear overdeterministic algorithm can be used to obtain these unknown coefficients, provided the values of the stresses are obtained at different points of coordinates (r, θ).

Values of the stresses obtained by optical measurement are processed with the linear overdeterministic algorithm in order to obtain the stress intensity factor. Since data may be collected in points near the crack tip, two or three terms from the series expansion would be sufficient for a good accuracy of the results. Nevertheless, seven terms were used for higher accuracy in this work.

For calculation purposes, the strain field is considered in a small quadratic area around the crack tip only, starting 0.3 mm after the crack tip and extending for 3 mm away from the tip. In this area, the strain values are read on a grid with 0.05 mm spacing, which guarantees a good degree of precision.

3. Numerical SIF determination

Linear elastic fracture mechanics (LEFM) can be used for the analysis of crack behaviour in damage tolerance. The fundamental postulate of LEFM is that the crack behaviour is determined solely by the values of the stress intensity factor, parameter which depends on the applied load and the geometry of the cracked structure.

Numerical methods should be primarily used for the stress analysis of engineering structures because of their complex geometry. When studying crack growth problems, the need for continuous re-meshing is a practical disadvantage of the finite element method (FEM).

The Boundary Element Method (BEM) is better suited for the incremental analysis of crack growth problems. An advantage of using the BEM method is the ability to easily model a great variety of cracks without the need for re-meshing of the model, as it would be necessary when using the finite element method. The main disadvantage of this method is related to difficulties of applying it to more complex structures.

The solution of general crack problems cannot be achieved with the direct application of the standard BEM, because the coincidence of the crack boundaries causes an ill-posed problem. For a pair of coincident source points on the crack boundaries, the algebraic equations relative to one of the points are identical with the algebraic equations relative to the opposite point, since the same boundary integral equation is applied at both coincident source points, with the same integration path, around the whole boundary of the problem. Among the techniques devised to overcome this difficulty, the most general are the sub-regions method and the dual boundary element method.

The DBEM introduces two independent boundary integral equations, with the displacement equation used for collocation on one of the crack surfaces and the traction equation used for collocation on the opposite crack surface. Consequently, general mixed-mode crack problems can be solved in a single-region boundary element formulation, with both crack surfaces meshed with the DBEM.

The stress intensity factor for the numerical verification is therefore calculated by the DBEM, using the J-Integral method. The code "Cracker" [8] is used for this purpose, since it has implemented a routine capable of predicting the crack growth path, which allows the validation of crack growth path predictions performed based on the optical strain measurements.

4. Comparison of numerical and experimental results

Figure 6 shows a comparison of experimental and numerical results of the strain near the crack tip for θ = 0º in front of the crack tip. An excellent agreement between the experimental and numerical result was found.

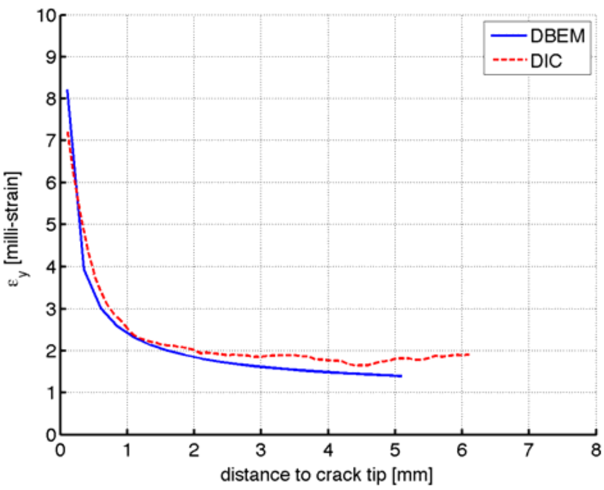

Figure 6. Strain Y comparison for numerical and experimental data.

For this case, only the stress field at the right side crack was determined experimentally and the stress intensity factor was compared to numerical results. Only mode I was considered, since the crack grows essentially perpendicularly to the loading direction. The experimental and numerical results are presented in Table 2. The numerical result was considered as reference value for the calculation of the difference parameter.

The strain field around the crack tip can be used in the present case to verify that the crack tip was correctly identified by the optical strain measurement technique. The good agreement shown in Figure 6 demonstrates the high confidence that is possible in this method. Nevertheless, the main problem found in this initial experiment was the correct determination of the crack tip in the images taken for digital image correlation.

Table 2. Comparison of the experimentally obtained SIF using DIC and the numerically determined SIF using the DBEM

a_2	DIC	DBEM	difference
SIF [MPa√mm]	475	487	2.5%

5. Discussion of the results and conclusions

As it can be seen, similar values have been obtained for the measured and calculated strain fields, which raises the confidence in the optical strain measurement system. Additionally, the experimental result for the SIF in mode I is promising, giving confidence that this technique can be applied to real life structures in order to assess their structural integrities.

Up to the moment, it is very difficult to precisely find the crack tip on the specimens, especially using digital images. This information is however fundamental for good results since the SIF strongly depends on this information. Therefore, a method has to be developed which guarantees that different operators can easily identify the crack tip.

A new procedure for processing the experimentally obtained strain values was proposed in this work. The strains measured with the GOM ARAMIS system were first converted into stresses, using the well-known equations of the theory of elasticity. Then, equations of the stress field around the crack tip, written as series development with seven terms, were used to fit the experimental data, obtaining thus an overdetermined system of equations in which the coefficients of the series expansion are the unknowns.

The overdeterministic algorithm, a numerical procedure for solving such systems, was used to solve the system and to obtain the stress intensity factor, which is the coefficient of the first term of the expansion.

The numerical calculations of the stress intensity factor using the dual boundary element method validated the methodology. A difference less than 3% was achieved, proving thus the reliability of the proposed method as long as the crack tip can be correctly detected.

Due to its flexibility, this method can be applied for in-situ fracture mechanics researches on real structures, for which a laboratory model is difficult or even impossible to conceive.

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